

K UbXYfi b['ni a 'A ci bh'K Y``]b[hcb. Fi bXk UbXYfi b['ni a ';]dZY`'XYg'A ci bh K Y``]b[hcb

9]b: Ø\fYfnia 'A ci bh'K Y``]b[hcb'<]_YžY]bYf
Fi bXk UbXYfi b['ni a 'A ci bh'K Y``]b[hcb'Gi a a]h'j cb
<cVUfh"6Y]b\U`hYhž'k Y`WXY'GhfYW_Yb'ni 'bY\a Yb'g]bXž
K UbXYf]bZcfa Uh]cbYb'i bX'a Y\f"'?i bUbm]'cXYf'A ci bh
K Y``]b[hcb']gh'XYf'a Uf_UbhY';]dZY`žXYf'U`g']_cb]gWXYf
<]bhYf[fi bX'ZØf'<cVUfhž'HUga Ub]Ybg'<Ui dhghUXhž'X]Ybh''
K À\fYbX'XYf';]dZY`a]h'XYa '5i hc'YffY]WXVUf']ghž'g]bX'X]Y
i b[`Ui V`]WXYb'5i gV`]W_Y'Ui Z'<cVUfh'j]Y`'VYggYfž'k Ybb'a Ub
g]Y'g]WX'j YfX]Ybh'' \DXYYgYa '6`c[VY]hfU['k YfXY']WX'X]Y
a Y]bYg'9fUWXhYbg'VYghY'K UbXYfi b['ni a 'A ci bh'K Y``]b[hcb
g_]nn]YfYb'''9g'\UbXY`h'g]WX'i a 'Y]bY'?ca V]bUh]cb'a Y\fYfYf
HfUW_gž'i a 'Y]bY'Fi bXgWX'Y]ZY'ni a 'D]bbUWY!;]dZY`i bX
ni fØW_'bUWX'H\Y'Gdf]b[g'ni 'V]'XYb'''K UbXYfi b['ni a ';]dZY`
XYg'A ci bh'/ Ua d\Y\``]d/

9]b: Ø\fYfini a 'A ci bh'K Y``]b[hcb'<]_YžY]bYf'Fi bXk UbXYfi b[ni a 'A ci bh'K Y``]b[hcb'Gi a a]h'j cb'<cVUfh"'6Y]b\U`hYhž'k Y`W\Y GhfYW_Yb'ni 'bY\a Yb'g]bXž'K UbXYf]bZcfa Uh]cbYb'i bX'a Y\f"

?i bUbm]'cXYf'A ci bh'K Y``]b[hcb']gh'XYf'a Uf_UbhY';]dZY`ž'XYf'U`g]_cb]gWXYf'<]bhYf[fi bX'ZØf'<cVUfhž'HUga Ub]Ybg'<Ui dhghUXhž X]Ybh"'K À\fYbX'XYf';]dZY``a]h'XYa '5i hc'YffY]WXVUf']ghž'g]bX'X]Y i b[`Ui V`]WXYb'5i gV`]W_Y'Ui Z'<cVUfh'j]Y``VYggYfž'k Ybb'a Ub'g]Y q]WX'j YfX]Ybh"

±b`X]YgYa '6`c[VY]hfU['k YfXY']W\'X]Y'a Y]bYg'9fUW\hYbg'VYghY K UbXYfi b['ni a 'A ci bh'K Y``]b[hcb'g_]nn]YfYb"'9g'\UbXY`h'g]W\ i a 'Y]bY'?ca V]bUh]cb'a Y\fYfYf'HfUW_gž'i a 'Y]bY'Fi bXgW\`Y]ZY ni a 'D]bbUWY!;]dZY``i bX'ni fØW_'bUW\'H\Y'Gdf]b[g'ni 'V]`XYb"

K UbXYfi b['ni a ';]dZY`'XYg'A ci bh K Y``]b[hcb

8]Y'K U\f\Y]h']ghžXUgg'Yg'j]Y`Y'A Ò[`]W_Y]hYb'[]VhžUi Z'XYb';]dZY`XYg'A ci bh'K Y``]b[hcb'ni 'k UbXYfb"=a ': c`[YbXYb'k YfXY']W\YXcW\XYb'K Y['g_]nn]YfYbž'j cb'XYa ']W\'[`Ui VYž'XUgg'Yf'XYfVYghY'K Y[']ghž'X]Yg'ni 'hi b"'8]YgY';]dZY`k UbXYfi b['ni a 'A ci bh K Y``]b[hcb']gh'Y]bY'Fi bXghfYW_Yž'k Ug'VYXYi hYhž'XUgg'G]Y'Ui Z'XYa FØW_k Y['b]W\h'Ui Z'XYa gY`VYb'K Y['ni fØW_[Y\Yb'a ØggYb"

Nia;]dZY`XYg'A ci bh'K Y``]b[hcb.

 $\label{eq:continuity} $H\Y'Gdf]b[g'2'@YbU\'JU``Ym'HfUW_'2'Gd\]bI'FcW_'2'@ckYfGUka]```HfUW_'2'Cf[Ub'D]dYg'HfUW_'2'N]['NU['HfUW_']$

 $\label{eq:continuous} J\ ca\ ;\]dZY``XYg`A\ ci\ bh`K\ Y``]b[\ hcb`bUW\`H\Y`Gdf]b[\ g.$

A ci bh'K Y``]b[hcb'Gi a a]h'2 'N]['NU['HfUW_'2 'D]bbUWY'HfUW_'2 H\Y'Gdf]b[g

K]Y ``Ub[Y 'XUi Yfh 'Ygž 'XYb 'A ci bh 'K Y``]b[hcb \]bUi Zni `Ui ZYb3'

8 U'H\Y'Gdf]b[g'VYfY]hg'Ui Z'Y]bYf'a]hh`YfYb'< Ò\Y'j cb'+&\$'A YhYfb ØVYf'XYa 'A YYfYggd]Y[Y```]Y[hž'XUi Yfh'X]Y'K UbXYfi b['Ui Z'XYb A ci bh'K Y``]b[hcb'fl%&+%'A YhYfŁ'bi f'Yhk U'%ž) 'V]g'&'Ghi bXYb" K Ybb'G]Y'YXcW\'ØVYf'Y]bY'UbghÀbX][Y':]hbYgg'j YfZØ[Yb'i bX'Y]b [i hYg'HYa dc'\U`hYb'a ÒW\hYbž'_Ubb'X]Yg']b'Yhk U'Y]bYf'Ghi bXY VYk À`h][h'k YfXYb"

? Ubbgh'Xi 'XYb'A ci bh'K Y``]b[hcb'\]bUi ZZU\fYb3'

8]Y'5bhk cfh']gh''U"'G]Y'_ObbYb'ØVYf'X]Y'D]bbUWY'FcUX'V]g'ni a;]dZY``XYg'A ci bh'K Y``]b[hcb'ZU\fYb"'CVYb'VYZ]bXYh'g]W\'Y]b [fc»Yf'DUf_d`Uhnž'Ui Z'XYa 'G]Y'cZh'j]Y`Y': U\fnYi [Y'UVghY``Ybž XUfi bhYf'[fc»Y'@]YZYfk U[Yb'i bX'6i gqY"

K Ybb'G]Y'U`gc'X]Y'i b[`Ui V`]W\Y'5i gg]W\h'Ui Z'<cVUfh'j ca ';]dZY` XYg'A ci bh'K Y``]b[hcb'gY\Yb'a ÒW\hYbž'UVYf'b]W\h'ni ': i » '[Y\Yb a ÒW\hYbž'gc``hYb'G]Y'a]h'XYa '5i hc'ZU\fYb"'6YUW\hYb'G]Y'YXcW\Ž XUgg'X]Y'GhfU» Y'ni a ';]dZY`'VY]'gW\`YW\hYa 'K YhhYf'i bX'GW\bYY cZh'Ub'j YfgW\]YXYbYb'GhY``Yb'[YgdYffh']gh''5a ': i » 'XYf'D]bbUWY FcUX'VYZ]bXYh'g]W\ Y]b'GW\]`Xž'XUg'G]Y'ØVYf'X]Y'U_h]j Yb GdYffi b[Yb']bZcfa]Yfh"

K Ybb'G]Y'_Y]b'Y][YbYg'5i hc'\UVYbž'_ObbYb'G]Y'Y]bY

; UbnhU[Yg_UfhY'ZØf'XYb'A ci bh'K Y``]b[hcb'9ld`cfYf'Vi WXYb"'8]Yg
]gh'Y]b'G\i hh`YVi g!GYfj]WYž'XYf'Ub'j YfgWX]YXYbYb'Di b_hYb
Ybh`Ub['XYf'D]bbUWY'FcUX'\A`hž'XUfi bhYf'H\Y'Gdf]b[gž'H\Y'7\U`Yh
i bX'D]bbUWY'Gi a a]h"

6i WXYb. K Y``]b[hcb'6ig'; UbnhU[Yg_UfhY

'U'h1 A CIBH'K 9@@B; HCB'; D: 9@61 G 2

@ŸB; GH9'K 5B89FIB; '5I: '89A 'A CIBH'K 9@\B; HCB. K Y``]b[hcb': U``g'i bX'XYf'j YfgWkk]bXYbXY'HUfb

8 Yf'A ci bh'K Y``]b[hcb'Gi a a]h'K U`_. Fi bXk UbXYfi b[

K UbXYfghfYW_Y.', ž+'? A 'FibXk Y[
8 Ui Yf.'&ž) '' 'Ghi bXYb 'FØW__Y\f'bUW\'H\Y'Gdf]b[g'
< O\Y.') -) 'A '; YgUa h\O\Ybi bhYfgW\]YX'p'\&+\%'A YhYf'5A G@'Ua
A ci bh'K Y``]b[hcb'Gi a a]h
GW\k]Yf][_Y]hq[fUX.'A]hhY`'fh]Ya `]W\'ghY]`Ł'

 $8]Y ;]dZY k UbXYfi b[ini a "A ci bh'K Y ``]b[hcbž'X]Y]Wk 'j cfgWk `U[Yž VY[]bbh']b "H\ Y "Gdf]b[g" "I a "\]Yf\ Yf'ni "[Y `Ub[Ybž'_ObbYb "G]Y "a]h XYa "5i hc "ZU\ fYbž'Y]b "H]W_Yh "ZØf XYb "K Y ``]b[hcb '9I d`cfYf'_Ui ZYb cXYf'j ca ": Yfb "HfYY "Ui g'ni ": I » "[Y\ Yb"$

8]Y'K UbXYfi b[']gh'n]Ya`]W\'ghY]`'i bX'ØVYfk]bXYh'Ui Z'Yhk U'(?]`ca YhYfb'ZUgh'*\$\$'<Ò\Yba YhYf"'8]Y'GhfYW_Yb'Ui Z'?i bUbm]'g]bX'YXcW\'i b[`Ui V`]W\'[i h'[YdZY[h'i bX'n]Ya`]W\'j c``"'8]Y'GhfYW_Y

U`h1 A C | B H K 9@B; HC B G | A A + K 5 B 8 9 F ? 5 F H 9 2

8]Y Ei Y``Yb ni a Gd\]bl FcW

BY\a Yb'G]Y'j cb'XYb'Gdf]b[g'Ui g'XYb'@YbU\'J U``Ym'HfUW_ž'XYbG]Y'Ui Z'XYf'fYW\hYb'GY]hY'XYf'D]bbUWY'FcUX'YffY]W\Ybž'i bX
[Y\Yb'G]Y'j cb'H\Y'Gdf]b[g'Ui g'XYb'A ci bh'K Y``]b[hcb'\]bUi Z''
8]Yg']gh'Y]bY'fY`Uh]j 'Z'UW\Y'GhfYW_Yž'X]Y'Ybh`Ub['XYg'?Ua a g'ni a Gd\]bI'FcW_'j Yf'Ài Zh''

U`h1 A C I B H 'K 9@=B; HC B 'K 5 B 8 9 F I B; 'N I A '; =D: 9@'5 I : '8 9 A @ 9 9 B 5 < 'J 5 @ 9 M H F 5 7 ? 2

Gd\]bl `FcW_`]gh'Y]b `[fc» Ufh][Yf'5i gg]Wk hgdi b_h'Ui Z'XYa `A ci bh K Y``]b[hcbž'XYf'À\b`]Wk Y'fk Ybb 'Ui Wk 'b]YXf][YfYŁ'5i gV`]W_Y'Ui Z <cVUfh'V]YhYh"'G]Y 'a ØggYb 'Y]b 'GhU\`hcf'ÒZZbYbž'i a j cfVY]ni _ca a Yb"'8Yf'F Uh']bghU``]YfhY'X]YgYg'Hcfž'i a '_`Y]bY ?]bXYf'XUfUb 'ni '\]bXYfbž'ni 'XYa 'ghY]`Yb'?`]ddYbUVZU```ni '[Y\Yb"

BUWXXYa 'G]Y'X]Y'5i gg]Wkh'[YbcggYb'\UVYbž'VYfY]hYb'G]Y' \Rightarrow fY 6Y]bY'Ui Z'Y]bYb'_i fnYbž'UVYf'gWkUfZYb'5Vgh]Y['Ui Z'XYa '@ck Yf GUk a]``'HfUW_'j cf"'G]Y'Z]bXYb'X]YgYb'5i g[Ub[gdi b_h'Ub'XYf : UVY`i b['X]fY_h'bYVYb'XYa '5i gg]Wkhgdi b_h'Gd\]bl'FcW_Ui g[YgWk]`XYfh"

U`h1 GD< +BL'FC7?'@CC?CIH'K 5B89FB'ACIBH K 9@@+B; HCB 2

U`h1 A C | B H 'K 9@@B; HC B ! K 5 B 8 9 F | B; 2

I `h]a Uh]j Y hUga Ub]gWX Y FY]gYfYggci fWY

** & 'Yd]gW\ Y '5_h]j]h\ hY b ']b 'HUga Ub]Yb'
\$\text{VYfbUW\ hi b[ga O[`]W_Y]hY b ']b 'HUga Ub]Yb 'f\ YghY'C fhY\ \text{Y' \&*}
YfghUi b `]W\ Y 'K UggYfZ\ ``Y']b 'HUga Ub]Yb '') 'A i gh!Xc!

J 9FK 5B8H9F 'DCGH.'&* 'hc``Y'8]b[Yž'X]Y'a Ub']b'<cVUfh HUga Ub]Yb'i bhYfbY\a Yb'_Ubb'

H\Y @ck Yf GUk a] ``HfUW_ i bX Cf[Ub D]dYg HfUW_

vYfei YfYb G]Y X]Y GhfU» Y i bX [Y\Yb G]Y k Y]hYf bUWk cVYbž V]g
G]Y Y]bY ; UVY i b[YffY]Wk Ybž X]Y 'U `g Cf[Ub D]dYg 'HfUW_
Ui g[YgWk]`XYfh]gh" 8]Yg]gh Y]b ``Ab[YfYf K Y[ž UVYf i a 'XYb';]d ZY`
XYg A ci bh K Y``]b[hcb ni 'YffY]Wk Ybž V]Y[Yb 'G]Y 'Ub 'X]YgYf
; UVY`i b[``]b_g UVž i a ']\a 'ni a 'N]['NU['HfUW_'ni 'Zc'[Yb"

 $8]Y \cdot Cf[Y \cdot dZY]ZYb \cdot g]bX \cdot Y]bYg \cdot XYf \cdot VYf \emptyset \setminus a hYghYb \cdot K \cup \{fnY]W \cdot Yb \cdot j \cdot cb < cVUfh'' \cdot 8]Yg \cdot g]bX \cdot \setminus c \setminus Y \cdot i \cdot bX \cdot ghY] \cdot Y \cdot 8 c \cdot Yf]hgÀi \cdot Yb \cdot ZX]Y \cdot Y]bYg \cdot XYf a Uf_UbhYghYb \cdot A Yf_a U \cdot Y \cdot j \cdot cb \cdot A ci \cdot bh \cdot K Y \cdot Y]b[hcb \cdot g]bX'' 5 bgW \cdot Y]bYbX \cdot k i fXYb \cdot X]YgY \cdot GÀi \cdot Yb \cdot]b \cdot XYf \cdot \Rightarrow fUnY]h \cdot [YV] \cdot XYh \cdot Z \cdot U \cdot g g]W \cdot HUga Ub]Yb \cdot j \cdot ca \cdot \setminus Yi \cdot h][Yb \cdot ? \cdot cb \cdot h]bYbh \cdot 5 bhUf_h]g \cdot hfYbbhY'' U \cdot h1 \cdot CF : 9 \cdot @D : 9 = 9B65 < B \cdot Z \cdot A CI \cdot BH \cdot K \cdot 9 \cdot @B : HCB \cdot K \cdot 5B8 \cdot 9FB \cdot 2 8 \cdot Yf \cdot Cf[Ub \cdot D]dYg \cdot HfUW_ \cdot V]YhYh \cdot bUW \cdot XYa \cdot j \cdot cfUb[Y[Ub[YbYb \cdot GhY] \cdot ghØW_ \cdot Y]bY \cdot [i \cdot hY \cdot J \cdot YfgW \cdot bUi \cdot ZdUi \cdot gY \cdot ZØf \cdot X]Y \cdot 6Y]bY'' \cdot BUW \cdot WU'' , $$ \cdot A YhYfb \cdot _ca \cdot a Yb \cdot G]Y \cdot Ub \cdot Y]bY \cdot k Y]hYfY : UVY \cdot i \cdot b[\cdot a \cdot]h \cdot XYa K \cdot Y[k \cdot Y]gYf \cdot ni \cdot a \cdot N][\cdot NU[\cdot HfUW_''$

N]W_!NUW_!HfUW_ ni a A ci bh K Y``]b[hcb Gi a a]h

U`h1 N]W!NUW!Gdif2

U`h1 N = !N5; !G7 < =9B9B < 5@H9F1B; 'K 9@eB; HCB 2 8]YgYf'`YhnhY'Ghc» 'V]YhYh'j]Y`Y'; Y`Y[Yb\Y]hYbž'Ubni\U`hYb'ibXX]Y'5i gg]W\h'Ui Z'A ci bh'K Y``]b[hcb'i bX' < cVUfh'ni 'VYk i bXYfb" 8 fØW_Yb'G]Y'YXcW\'k Y]hYfž'XYbb'X]Y'VYghYb'5i gg]W\hYb'\UVYb G]Y'XYZ|b|h|j'j ca';]dZY`°

U`h1 A C I B H 'K 9@ \oplus B; HC B 'K 5 B 8 9 F I B; ž'G I A A \oplus H'HF 5 7? ž'N \doteqdot N 5; 'HF 5 7? 2

A ci bh'K Y``]b[hcb'Gi a a]h'/ 'H\Y'D]bbUWY @cc ci h

8 UgʻYfghYʻNY]WXYbžʻXUgʻG]YʻgY\Ybʻk YfXYbžʻXUggʻG]Yʻg]WXʻXYa;]dZY``bÀ\Yfbžʻ]ghʻX]Yʻ\cWXʻUi ZfU[YbXYʻGd]hnYžʻX]YʻUʻg Ui ghfU`]gWXYfʻGYbXYhi fa 'VY_Ubbhʻ]gh"ʻK YbbʻG]Yʻj cbʻ\]YfʻUi g k Y]hYf[Y\Ybžʻ[Y`Ub[YbʻG]Yʻni a 'cZZ]n]Y``Ybʻ;]dZY`di b_hii bX'ni Y]b][Ybʻ5i gg]WXhgdi b_hYb"

U`h1 ; =D: 9@89G'ACIBH'K 9@@B; HCB 2

8 Ug 6 Y c V U W hi b [gn Y b h fi a

8 UgʻD]bbUWYʻC VgYfj Uh]cbʻ7 YbhYfʻ]ghʻY]bʻ ⊕ Xccf! 5 i gg]Wk hgdi b_h a]hʻj]Y`Ybʻ8 YhU]`gʻØVYfʻX]Yʻ; YgWk]Wk hYʻXYgʻ6 Ui gʻXYfʻD]bbUWY FcUX"ʻ9gʻ]ghʻY]bʻ[fc» Ufh][YfʻCfhži aʻXYaʻK YhhYfʻni 'YbhZ]Y\Yb i bX`X]Yʻ5 i gg]Wk hʻj cbʻcVYbʻni 'VYk i bXYfb"

K Ybb 'G]Y 'Ua 'NYbhfi a 'j cfVY]'k Y]hYf[Y\Ybž'[]Vh'Yg 'Ui Z'Y]bYf VfY]hYb 'i bX '[YfÀi a][Yb 'Dfca YbUXY 'Ui W 'Y]b 'dUUf 'Yd]gW Y 5i gg]W hgdi b_hY 'a]h'6`]W_'Ui Z' < c VUfh" U`h1 < C65F H'J CA 'A CIBH'K 9@B; HCB 2 U`h1; D: 9@K 5B89FIB; '5A 'A CIBH'K 9@B; HCB 2

5 i gg]W hgd `UhhZcfa `GØXk Ygh

8]Y'nk Y]hY'5i gg]Wkhgd`UhhZcfa 'Ui Z'XYa';]dZY`'XYg'A ci bh K Y``]b[hcb']gh'X]Y'Gci h\'K Ygh'J]Yk]b['D`UhZcfa "'8]Yg']gh'Y]bY k Y]hYfY'_i fnY'Dfca YbUXY'a]h'6`]W_'Ui Z'XUg'<]bhYf`UbX'j cb K Y``]b[hcb'DUf_'i bX'X]Y'XU\]bhYf'`]Y[YbXYb'6Yf[Y"

FØW__Y\f'bUW\'Gdf]b[g'ØVYf'XYb'N]['NU['HfUW_'/D]bbUWY'HfUW_

 $8 \ Yf \ YhnhY \ 5 \ VgW. \ b]hh \ XYf \ K \ UbXYf \ b[\ ni \ a \ A \ ci \ bh \ K \ Y \]b[\ hc b \ ZØ \ fh \ ØVYf \ XYb \ N][\ NU[\ HfUW_i \ bX \ XYb \ D]bb \ UWY \ HfUW_ini \ fØW_b \ bUW \ H \ Y \ Gdf]b[\ g" \ 8 \ Yf \ Y]bn][\ Y \ F ØW_k \ Y[\ Ui \ Z'X]YgYf \]dZY \ fi \ bXY \]gh \ XYf \ N][\ NU[\ HfUW_ž \ XU \ Yg \ Y]bY \ U \ hYfb \ Uh]j \ Yb \ HfU] \ g'ni \ a \ ; \]dZY \ [\]Vh''$

GcVU`X'G]Y'Ub'XYf'; UVY`i b['Ub[Y_ca a Yb'g]bXž'k c'G]Y'XYb'N][
NU['HfUW_'j ca 'Cf[Ub'D]dYg'HfUW_'Yf\U`hYb'\UVYbž'ZU\fYb'G]Y
[YfUXYUi g'k Y]hYf'i bX'Zc`[Yb'G]Y'XYb'K Y[k Y]gYfb'ni a 'H\Y
D]bbUWY'HfUW_"

8]YgYf'K Y[']gh'j]Y`'gUbZhYf'U`g'XYf'GÀ[Yk Yf_gk Y['i bX']bg[YgUa h Y]b'VYei Ya Yf'5Vgh]Y['ni fØ \underline{W} 'bUW\'H\Y'Gdf]b[g"

$K Y]hYfY^5_h]j]hAhYb^]b^XYf^BA\Y^jcb^<cVUfh$

 $K \ Ybb = bYb \times YgYf @Y hZUXYb \times ZØf \times YYa cibh K Y ``]b[hcb! \\ K \ UbXYfib[ibX \times XYb ;]dZY `gdUn]Yf[Ub[i[YZU ``Yb \ Uhž `YgYb G]Y ibVYX]b[hY]b][Y a Y]bYf `UbXYfYb 6 `c[!Dcghg \ ZØf `HUga Ub]Yb " IbhYb \ UVY \]WX \ Y]b][Y `Ui Z[Y `]ghYhž j cb \ XYbYb \]WX \ g]WX Yf \ V]b \ Z XUgg G]Y \ ig]WX j cb \ FY]gYb \ ibX \ 5VYbhYi Yfb \]bgd]f]YfYb ``UggYb \ k YfXYb \ `$

п

8 Y hU1`a

6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. 'Y]bY!fY]gY "XY