

Hcd!Hcif]gaign]Y`Y`]b`J]YhbUa. 9bhXYW_Yb`G]Y`X]Y`VYghYb`8ÒfZYf°

9bhXYW_Yb'G]Y'XYb'`ÀbX`]W\Yb'Hcif]gaig']b'J]YhbUag NYbhfU`fY[]cb.'Y]bn][Ufh][Y'?i`hifYbž'k]fhgW\UZh`]W\Y 7\UbW\b'ibX'Uig[YnY]W\bYhY'8\OfZYf"



HUb' < cU'J]``U[YžEiUb['6]b\žJ]YhbUa'!8]Y``ÀbX`]W\Yb; YV]YhY`XYf`NYbhfU`fY[]cb'J]YhbUa g'fØW_Yb'j YfghÀf_h']b'XYb

: c_i g'XYg'bUW\U`h][Yb'Hci f]ga i g''8]YgY`FY[]cbYb'g]bX`VY_Ubbh

ZØf']\fY'Y]bn][Ufh][Yb'_i `hi fY``Yb'i bX`bUhØf`]W\Yb'9fV[ØhYfž'X]Y

g]W\XYi h']W\'j cb'ghÀXh]gW\Yb'5hhfU_h]cbYb'i bhYfgW\Y]XYb''

♣Zcfa Uh]cbYb'j cb'J]YhbUa "j b'nY][Ybž'XUgg'XYf``ÀbX`]W\Y

Hci f]ga i g'b]W\h'bi f'ni f'gcn]cÒ_cbca]gW\Yb'9bhk]W_`i b[

VY]hfÀ[hz'gcbXYfb'Ui W\'cZh']b'UV[Y'Y[YbYb'; YV]YhYb'_Y]bfÀi a][
Ui g[Yf]W\hYh']gh''8UVY]']gh'X]Y'JYf_Y\fg]bZfUghfi_hi f'VY[fYbnhz'i bX'Yg'a Ub[Y`h'Ub'♣j Ygh]h]cbga]hhY`b'gck]Y'Ub'[i h'[YgW\i`hYb

<i a UbfYggci fWyb''5i W\X]Y'♣Zcfa Uh]cbghYW\bc`c[]Y']gh'cZh'b]W\h

a]h'XYb'U_hi Y``Yb'A Uf_hVYXØfZb]ggYb'gmbW\fcb]g]Yfhz'k Ug'X]Y

5bk YfVi b['i bX'6]bXi b['j cb'Hci f]ghYb'YfgW\k Yfh''

8 Y b b c W 'k i f X Y b 'X i f W 'X Y b '6 Y g W `i g g 'B f"' - & & #E 8 ! H H ['j ca '&" 5 i [i g h & \$ & A U » b U \ a Y b 'n i f': Òf X Y f i b ['X Y g `` À b X `] W Y b H c i f] g a i g '] b 'X Y f 'N Y b h f U `f Y [] c b 'Y f [f] Z Z Y b "'; Y a Y] b X Y b '] b D f c j] b n Y b 'k] Y 'E i U b ['6] b \ ž' < i Y ž' E i U b ['B U a ž' E i U b ['B [U] 'i b X ? \ U b \ ' < c U \ U V Y b 'I a g Y h n i b [g d `A b Y 'Y b h k] W Y `h ž' i a 'h c i f] g h] g W Y N] Y `Y 'n i 'Z Òf X Y f b " 6 Y g c b X Y f g `\ Y f j c f n i \ Y V Y b 'g] b X 'H U b ' < c U 'J] `` U [Y] b 'E i U b ['B U a ž' X] Y V Y] X Y 'U `g 'V Y g h Y 'H c i f] g h Y b X Òf Z Y f 'U i g [Y n Y] W \ b Y h 'k i f X Y b "

5 i gnY]WX bi b[Yb ZØf \YfUi gfU[YbXY 8 OfZYf

8]Y`I BK HC`\Uh`gY]h`&\$&%`XYb`6Ygh`Hcif]ga `J]``U[Yg`5k UfX`]bg @YVYb`[Yfi ZYbži a ``ÀbX`]W\Y`8 OfZYf`ZØf`]\fY`bUhØf`]W\Y GW\Ob\Y]hž`]\f'_i `hi fY``Yg`9fVY`i bX`]\fY`bUW\\U`h][Yb Hcif]ga i gdfU_h]_Yb`ni `k ØfX][Yb"`J]YhbUa `_Ubb`ghc`n`Ui Z`XfY]Ui g[YnY]W\bYhY`8 OfZYf`gY]b.`H\U]`<U]`f&\$&&\z`HUb`<cU`f\\$\$&' \text{LibX`HfU`E i Y`f\\$\$&(\text{z`k }Y`\U\\\Ui\]\YhbUa "cf[`VYf]W\hYh"

HUb'<cU'J]``U[Yž'[Y`Y[Yb']b'XYf'Dfcj]bn'EiUb['6]b\ž'\Uh'g]W\
XifW\gY]bY'UhYaVYfUiVYbXYb'<Ò\`YbgmghYaYž'k]Y'XUg'Hi'@Ub!
GmghYaž'Y]bYb'BUaYb'[YaUW\h''<]Yf'VYkU\fYb'X]Y']bX][YbYb
B[icb'AYbgW\Yb']\fY'HfUX]h]cbYbž'ibhYf'UbXYfYa'VY]a'<c
H\icW7U'fJc`_g[YgUb[Ł'ibX'XYa':Ygh'FUa'H\Ub['6U"'8]Y
9]bZØ\fib['Y]bYg'kYhhYfUXUdh]jYb'Hcif]gaigacXY``g'\Uh'XYa
8cfZ'&\$&&^[Y\c`ZYbž'Y]bYb'FY_cfX'jcb', "\$\$%'6YgiW\Yfb'nijYfnY]W\bYb"

HfU'E i Y'J Y[YhUV`Y'J]``U[Y'i bX'gY]bY bUW\\U`h][Yb'=b]h]Uh]j Yb

 $\label{thm:linear} HfU'EiY'JY[YhUV'Y'J]``U[Yž'bif'' '_a 'bÒfX`]WX'jcb'<c]'5b\\ [Y`Y[Ybž'\Uh'g]WX'YVYbZU``g'Y]bYb'VYgcbXYfYb'D`Uhn']b'XYf hcif]gh]gWXYb'@UbXgWXUZh'[Yg]WXYfh''A]h'ØVYf'&$$; Ya ØgYUbVUi\Uig\U`hYb'ibX'Y]bYf': `ÀWXY'jcb'%, '<Y_hUf'\UhXUg'8cfZ'&$&&'XYb'GhUhig'Y]bYg'BUh]cbU`Yb'=a a UhYf]Y``Yb?i`hifYfVYg'Yf\U`hYb'''Hcif]ghYb'_\ObbYb'\]Yf']b'X]Y'KY`h'XYg; Ya ØgYUbVUig'Y]bhUiWXYb'ibX'``c_U`Y'DfcXi_hY'YfkYfVYb'''}$

Ni XYa "V]YhYb"&' "6Y\YfVYf[i b[gVYhf]YVY"i bX"%* "FYghUi fUbhg $5fVY]hgd `AhnY 'ZØf 'X]Y 'Ofh`]WXY '6Yj O`_Yfi b["$

± E i Ub[B Ua \UVYb ØVYf %&, 5 [fUfhci f]ga i g!GhUbXcfhY ØVYf %"\$\$\$ < Ui g\U`hY Ub[Ync[Ybži bX Yg k]fX [YgW Àhnhž XUgg X]Y 9]bbU\a Yb Ui g XYa ; Ya Y]bgW UZhghci f]ga i g ZØf &\$&(ØVYf %) \$ A]``]UfXYb J B 8 VYhfU[Yb k YfXYb "6YgcbXYfg \Yfj cfni \YVYb]gh \]YfVY] XUg 6Um A Ui 7 c Wcbi h: cfYgh]b 7 Ua H\Ub\ž XUg]a → 从 f &\$&(ØVYf % A]``]cb 6Ygi W Yf Ub`c W_hY i bX 9]bbU\a Yb j cb ' \$ A]``]UfXYb J B 8 [YbYf]YfhY "

9]b'k Y]hYfYf'@]W\hV`]W_']gh'X]Y'9bhk]W_'i b['j cb'HUb'<cU'ni 'Y]bYa [Ya Y]bgW\UZh`]W\Yb'Hci f]ga i gXcfZž'XUg'i bhYf'UbXYfYa <ca YghUmgž'`c_U`Y'FYghUi fUbhg'i bX'_i `hi fY``Y'Hci fYb'UbV]YhYh'' 8 Ug'l bhYfbY\a Yb'7\i U'A Y'8 Uh'fC | U`]gŁ'\Uh'gY]h'&\$%(5VYbhYi Yfhci fYb']a 'Hi '@Ub!<Ò\`YbgmghYa ']b]h]]YfhY'i bX a]hh`Yfk Y]`Y'%&\$'`c_U`Y'5fVY]hg_fÀZhY'VYgW\ÀZh][hž'XYfYb a cbUh`]W\Yg'9]b_ca a Yb'g]W\'nk]gW\Yb'* i bX', 'A]``]cbYb'JB8 VYk Y[h''

5b[Yg]Wkhg'X]YgYf'9fZc`[Y'nY][h'g]Wkž'XUgg'XYf'`ÀbX`]WkY
Hci f]ga i g']b'J]YhbUa '`Ub[Zf]gh]['gck c\`'X]Y'k]fhgWkUZh`]WkY'@U[YU`g'Ui Wk'XUg'_i `hi fY``Y'6Yk i gghgY]b'XYf'; Ya Y]bXYb'ghÀf_Yb_Ubb"'A]h'XYa ': c_i g'Ui Z'bUWk\U`h][Y'DfU_h]_Yb'i bX'X]Y
9]bVYn]Y\i b['XYf'Òfh`]WkYb'6Yj Ò`_Yfi b['_ObbYb'X]YgY'`ÀbX`]WkYb; YV]YhY'k Y]hYf\]b'U`g'UhhfU_h]j Y'FY]gYn]Y`Y'k UWkgYb'i bX'g]WkYbhk]W_Y`b"

8 Ugʻ9b[U[Ya YbhʻZØfʻXYbʻ`ÀbX`]WXYbʻHci f]ga i gʻk]fX'Uaʻ-"
8 YnYa VYfʻ&\$&(ʻVY]ʻXYfʻnk Y]hYbʻA\f`]WXYbʻJYfgUa a `i b[ʻXYf
I BK HC gʻ6YghʻHci f]ga ˈJ]``U[Ygʻ]bʻ<c]ʻ5bʻk Y]hYfʻX]g_i h]Yfhžʻk c
bUWX\U`h][Yʻi bXʻ[Ya Y]bgWXUZhgcf]Ybh]YfhYʻHci f]ga i gdfU_h]_Yb
la ʿA]hhY`di b hʻghY\Ybʻk YfXYb"

8 Y hU]`g	
Cfh	HUbʻ <cuʻj]``u[yžeiub[ʻ6]b\žj]yhbua< td=""></cuʻj]``u[yžeiub[ʻ6]b\žj]yhbua<>

8 YhU]`g	
EiY``Yb	• k k k "j]Y hb Ua "j b
	• U\Uj]YhbUa "cf[

6Ygi WXYb'G]Y'i bg'Ui Z.'Y]bY!fY]gY"XY