

$5]fVbV!9h]_YhhY. % H]ddg ZØf; ÀghY$

Jcb XYf [Ya Y]bgUa Yb Bi hni b[j cb 6UXYn]a a Yfb V]g \]b ni a 'Hf]b_[Y`X ZØf; Ugh[YVYf Yf_`AfYb k]f X]Y f]Wkh][Y 5]fVbV!9h]_YhhYž XUa]h G]Y Y]b [fc > Ufh][Yf; Ugh gY]b _ObbYb K A\fYbX i bgYfYf FY]gY i a 'X]Y K Y`h d`UbYb k]fž X]Y : Yf]Ybk c\bi b[ggY]hY 5]fVbV ZØf Y]bYb [fc > Yb HY]`i bgYfYf I bhYf_ØbZhY ni bi hnYb" =\W V]b gY]h ZUgh XfY] =\U\fYb \; Ugh[YVYf i bX \UVY Y]b ZY]bYg; Ygd Øf XUZØf Ybhk]W_Y`hž k Ug Y]bY [i hY 5]fVbV!9h]_YhhY Ui ga UWkh" <]Yf hY]Y]W '%\$ k]Wkh][Y H]ddgž XUa]h G]Y Y]b [fc > Ufh][Yf; Ugh gY]b _ObbYb" %" @YgYb G]Y X]Y [YgUa hY @]ghY i bX bi hnYb G]Y XYb ? U`YbXYf '; Ugh[YVYf \Y`ZYb; AghYb [YfbY XUVY]ž FYghUi fUbhg cXYf 5_h]j]hAhYb ni 'Z]bXYbž X]Y]\fYa ; YgWa UW / Ua d \(\famour\) Y`]d/

KÀ\fYbX'i bgYfYf'FY]gY'i a 'X]Y'K Y`h'd`UbYb'k]fž'X]Y
: Yf]Ybk c\bi b[ggY]hY'5]fVbV'ZØf'Y]bYb'[fc»Yb'HY]`'i bgYfYf
I bhYf_ØbZhY'ni 'bi hnYb"

%"'@YgYb'G]Y'X]Y'[YgUa hY'@]ghY''i bX'bi hnYb G]Y'XYb'?U`YbXYf° ; Ugh[YVYf`\Y`ZYb`; ÀghYb`[YfbY`XUVY]žFYghUi fUbhg`cXYf 5_h]j]hÀhYb`ni `Z]bXYbž'X]Y`]\fYa `; YgW\a UW_`YbhgdfYW\Yb"`K]f a ÒW\hYbž'XUgg`G]Y`Y]bY`hc``Y`NY]h`]b`i bgYfYf`GhUXh`j YfVf]b[Yb i bX`bUhØf`]W\`a ÒW\hYb`k]fž'XUgg`G]Y`i bg`k]YXYf`VYgi W\Yb°

K Ugʻk]f`\UggYbžʻ]ghžkYbbʻ6Ybi hnYfʻi bgʻY]bYʻBUW\f]W\h'a]h'XYf
: fU[Yʻ=ghʻ=\fʻD`Uhnʻj YfZØ[VUf3 ˈgYbXYb"cXYfʻ;]Vh'YgʻÒZZYbh`]W\Y
JYf_Y\fga]hhY`3 ˈcXYfʻ k]YʻkY]hʻ]ghʻXYfʻ6U\b\cZ3 ˈcXYf
]f[YbXYhk UgžʻXUgʻXi fW\'Y]bYbʻZØW\h][Ybʻ6`]W_ʻUi ZʻX]Y
5i Z]ghi b[gVYgW\fY]Vi b[ʻVYUbhk cfhYh'k YfXYbʻ_Ubb"

K]f`\UVYb`NY]hii bX`A Ø\Y`XUfUi Zij Yfk YbXYhžii bgYfY 6YgW\fY]Vi b[Yb`ni gUa a Ybni ghY``Ybžii bX`XYf`? U`YbXYf`]gh'[YbUi Xcfhž'gcXUgg`X]YgY`5fh'j cb`: fU[Yb`nY]hfUi VYbX`i bX`Zfi ghf]YfYbX gY]b`_Ubb"

K Ybb'G]Y' ♣ Zcfa Uh]cbYb'VYbÒh][Ybž'X]Y'b]W\h']b'XYf
6YgW\fY]Vi b['Ybh\U`hYb'g]bXž'gYbXYb'G]Y'i bVYX]b[h'Y]bY
BUW\f]W\h'Ub'XYb'; Ugh[YVYfž'UVYf'`YgYb'G]Y'V]hhY'ni Yfgh'X]Y
5i Z]ghi b["

 $"U"h1 = 5]fVbV!9h]_YhhY!H]ddg!; Ugh 2$

$\&"'9]bZ\emptyset\fib[gVchgWXUZhYb.'hib'cXYf'b]WXhhib3$

K Ybb 'G]Y 'Y]b 'j c``ghÀbX][Yg 'DfcZ]``i bX 'Y]bY 'cXYf'nk Y] '[i hY 6Yk Yfhi b[Yb '\UVYbž'a ØggYb 'G]Y 'j cf'XYf'6i WXi b['_Y]bY 9]bZØ\fi b[gbUWXf]WXh'gYbXYbž'Yg 'gY]'XYbbž'XYf'; Ugh[YVYf j Yf`Ub[h'X]Yg 'Ui gXfØW_`]WX ']b 'XYf'6YgWXfY]Vi b['XYg' \Delta gYfUhg"

K Ybb 'Yg 'X]f 'k]f_`]W\ 'dY]b`]W\ ']ghž 'VY] 'Ya UbXYa 'ni 'V`Y]VYbž 'c\bY j cf\Yf' < OZ]W_Y]hYb 'Ui gni hUi gW\Ybž 'XUbb ']gh 'XUg ']b 'C fXbi b[ž UVYf 'XYb_Y 'b]W\ hž 'XUgg 'Xi 'Yg 'U`g 'gY`Vghj Yfgh\bX`]W\ 'hi b 'a i ggh''

 $GhY``Yb`G]Y`ghUhhXYggYb`Y]bY`6i Wib[gUbZfU[Y`ibX`ZØ[Yb`G]YY]bY`Y]bYY]bYY]bYBch]n`\]bni "`8]Yg`k]fX`ZØf`X]Y`aY]ghYb`<cghgUigfY]WXYb"$

' "'6]hhYb'G]Y'b]W\h'i a '6UfnU\`i b[

 $\label{eq:localized-substance} $$ \frac{1}{2} W^k k Y^* \tilde{z}^2 X Ugg^* X^* Y^* \tilde{z}^2 Y^*$

 $8]Y 5]fVbV!; YV\emptyset \f'X]Ybh'XYa 'GW\i hn'j cb'; ÀghYb'i bX ; Ugh[YVYfb'gck c\`']b'6Yni ['Ui Z'X]Y'; Y`Xj Yfg]W\Yfi b['U`g'Ui W\Ui Z'X]Y'dYfgÒb`]W\Y'G]W\Yf\Y]h''' < U`hYb'k]f'i bg'Y]bZUW\''Ub'X]Y <math display="block">FY[Y`b\check{z}`c_Um3$

("'6i WXYb'G]Y'gWXbY```bUWX' \Rightarrow fYf'5bZfU[Y

 $GWX^{}]Y \gg Yb^{}G]Y^{} \triangleq fY^{}6i WXib[^{}gc^{}gWXbY^{}Xk]Y^{}a \tilde{O}[^{}]WX^{}UV''^{}: \emptyset f'ibg \\]gh^{}Yg^{}cZZYbg]WXh^{}]WXZ^{}k Ybb^{}; \tilde{A}ghY^{}hcbbYbk Y]gY^{}9!A U]^{}g^{}Ub^{}j]Y^{}Y \\ ; Ugh[^{}YVYf^{}[YgWX]W_h^{}VUVYb^{}ibX^{}UiZ^{}XUg^{}VYghY^{}5b[YVch^{}k UfhYb''] \\$

K]f'j YfghY\Yb'i bX'gW\ÀhnYbž'XUgg'G]Y'K Yfh'Ui Z'Y]b'[i hYg'DfY]g!
@Y]ghi b[g!JYf\À`hb]g'`Y[Ybž'UVYf'k Ybb'k]f'Y]b][Y'HYfa]bY'ZØf'G]Y
ZYgh\U`hYbž']gh'Yg'i b\ÒZ]W\ž'i bg'\Àb[Yb'ni '`UggYb"

) "'=bZcfa]YfYb'G]Y'=\fYb'; Ugh[YVYfž'k Ybb G]Y'g]WX'j YfgdÀhYb

J Yfgi WX Yb 'G]Yž' → fYa '; Ugh[YVYf'Y]bY '[YbUi Y'J cfghY``i b['XUj cb ni '[YVYbž'k Ubb 'G]Y 'Ub_ca a Yb 'k YfXYb"'G]Y '\UVYb a Ò[`]WX Yfk Y]gY 'D`ÀbYž'X]Y 'g]Y 'i a 'G]Y '\Yfi a 'UbdUggYb'a ØggYbž cXYfž'k Ybb 'g]Y 'b]WX h'Ua 'gY`VYb'C fh'V`Y]VYbž'a ØggYb'g]Y 'ni ']\fYa

, VYfbUW\hi b[gcfhfY]gYb"

 $K Ybb `G]Y `U`gc `[YgU[h' \setminus UVYb \check{z}'XUgg `G]Y `i a `\&\$.\$\$'I \setminus f`Ub_ca a Yb k ØfXYb \check{z}'i b X `Yg `hUhg AWA`]WA `gc `Ui gg]Y \setminus h\check{z}'U`g `c V `Yg `&&.\$\$'I \setminus f`gY]b k]fX\check{z}'hY]`Yb `G]Y `X]Yg `\= fYa `; Ugh[YVYf `i b VYX]b[h`a]h'' 9b hgWA i `X][Yb `G]Y `X]Y `J Yfn O[Yfi b[`i b X ``UggYb `G]Y `g]Y `k]ggYb \check{z} XUgg `G]Y `]\fY': `YI]V]`]hAh `gWA AhnYb''$

* " 8 Yb $_{Y}$ b G Y X UfUb X XUgg Y g Y Y]b Y

#W 'k Y]» 'b]Wk hž'k]Y 'Yg'Yi Wk '[Y\hž'UVYf'k Ybb']Wk ']b'Y]bYa '<chY`
V]bž'`Ui ZY']Wk ']b'a Y]bYf'FcVY'\Yfi a 'i bX'ZØ\`Y'a]Wk 'k]Y'Y]b
? Òb]["'5``Yg']gh'a U_Y``cgž'k Ybb']Wk 'Ub_ca a Y'i bX'k Yb][Yfž'k Ybb
]Wk '[Y\Y"' + Y]bYf'5]fVbV!G]hi Uh]cb'a ØggYb'G]Y'YXcWk 'XUfUb
XYb_Ybž'XUgg'G]Y'g]Wk 'VY]''Ya UbXYa 'ni '<Ui gY'VYZ]bXYb"</pre>

8 UgʻVYXYi hYhžʻXUggʻUŁʻb]W\h'U``Ygʻgcʻa U_Y``cgʻ]ghʻk]Yʻ]bʻY]bYa < chY`žʻk Y]``XcfhʻhUhgÀW\`]W\`A YbgW\Ybʻk c\bYbʻi bXʻVŁ'Ygʻ_Y]bN]a a Yfa ÀXW\Ybʻ[]VhžʻXUgʻXYbʻ; ÀghYbʻ\]bhYf\Yfʻdi hnh"

K]f'Yfk UfhYb'g]W\Yf']W\'b]W\hž'XUgg'G]Y'X]Y'K c\bi b['VY]' \Rightarrow fYf 5VfY]gY'fY]b][Ybž'UVYf'Y]b'Ub[Ya YggYbYg'A U»'Ub'GUi VYf_Y]h k]fX'[YgW\Ahnh'']bgVYgcbXYfYž'k Ybb' \Rightarrow bYb'_Y]bY FY]b][i b[g[YVØ\f'VYfYW\bYh'k i fXY"

+"": fU[Yb'G]Y'bUWX'Y]bYa '6UXYn]a a Yfd`Ub

K Ybb'G]Y'g]W\'Y]b'<Ui g'hY]`Ybž'UbghUhh'Y]b'[UbnYg'<Ui g#K c\bi b[ni 'Vi W\Ybž'_Ubb'Yg'gY]bž'XUgg'G]Y'Ua '9bXY'Y]b'6UXYn]a a YfhY]`Yb"' = X]YgYa ': U``'[Y\Òfh'Yg'ni f'[i hYb'5]fVbV!9h]_YhhYž'ni ZfU[Ybž'cV'XYf'; Ugh[YVYf'Yg'ni 'VYgh]a a hYb'NY]hYb'VfUi W\h''

8]Y '7\Ub\W\b'ghY\Yb'[i hž'XUgg'g]Y 'Uf\Y]hYbž'k À\fYbX'G]Y']a
I f`Ui V'g]bX"'K Ybb'g]Y'U`gc'a cf[Ybg'Y]bY'\U`VY'Ghi bXY'j cf'XYf
5fVY]h'Ui Z'X]Y'Hc]`YhhY'a ØggYbž'k YfXYb'g]Y'Yg'gY\f'ni 'g\W\AhnYb
k]ggYbž'k Ybb'G]Y'Yg']\bYb'ØVYf`UggYb"

, "'GU[Yb'G]Y'XYa'; Ugh[YVYfž'k Ybb'Yhk Ugb]W\h'gh]a a h

K Ybb'G]Y'a]h'Yhk Ug'i bni Zf]YXYb'g]bXž'hY]`Yb'G]Y'X]Yg'XYa ; Ugh[YVYf'gc'ZfØ\ 'k]Y'a O[`]W 'a]h''8]Y'a Y]ghYb'j cb'i bg'k YfXYb g]W\ 'bUW\ '\]bhYb'VYi [Ybž'i a 'i bgYfYb'; ÀghYb'ni '\Y`ZYbž'U`gc gY]Yb'G]Y'V]hhY'Y\f`]W\ ž'k Ybb'Yhk Ug'b]W\ h'gh]a a h'''K Ug'G]Y'b]W\ h hi b'gc``hYbž']ghž'ni 'Y]bYa 'DfcV`Ya 'ni 'gW\ k Y][Yb'i bX'Yg'bi f']b \Rightarrow fYf'6Yk Yfhi b['bUW\ 'XYa '5i ZYbh\U'h'ni 'Yfk À\bYb''

- "'BY\a Yb'G]Y'FØW_g]W\h'Ui Z'ni gÀhn`]W\Y; ÀghY

K Ybb'G]Y'Y]b'N]a a Yf'ZØf'Y]bY'DYfgcb'Vi W\Ybž'[Y\Yb'G]Y'b]W\h XUj cb'Ui gž'XUgg'Ya UbX'UbXYfYg'VY]' \Rightarrow bYb'ØVYfbUW\hYb'_Ubb']bgVYgcbXYfYž'k Ybb'G]Y'XUg'<Ui g'a]h' \Rightarrow fYa '; Ugh[YVYf'hY]'Yb"

K Ybb 'Yg 'g]W 'i a 'Y]bYb '`Àb[YfYb '5i ZYbh\U`h'\UbXY`h'i bX 'G]Y 'g]Y _YbbYb[Y`Yfbh'\UVYbž']gh'Yg 'j]Y ``Y]W\h']b 'C fXbi b['ni 'ZfU[Ybž'cVg]Y 'Yhk Ug 'XU[Y[Yb'\UVYbž'XUgg 'G]Y 'Y]bYb'; Ugh'Y]b`UXYb"

6Y]'Y]bYa '_i fnYb'5i ZYbh\U`h']gh'XUg'U``YfX]b[g'Yhk Ug'ZfYW\ž ni a U``j]Y`Y'; Ugh[YVYf'ZØf'Y]bYb'nk Y]hYb'; Ugh'j Yf`Ub[Yb"

%\$"'<]bhYf`UggY'Y]bY'6Yk Yfhi b['fgW\bY``Ł

8]Ygʻ]ghʻc ZZYbg]W\h`]W\žʻUVYfʻk YbbʻY]bʻ; Ugh[YVYfʻgY]bʻNi \Ui gY ZØfʻG]Yʻ[YÒZZbYhʻ\UhžʻgW\ÀhnhʻYfʻYgʻgY\fžʻk YbbʻG]Yʻ_i fnʻbUW\ \Rightarrow fYa '5i ZYbh\U`hʻY]bYʻ6Yk Yfhi b['UV[YVYb"'J Yfgi W\YbʻG]Yžʻa Y\f U`gʻY]bYʻNY]`Yʻni '\]bhYf`UggYbʻi bX`[YVYbʻG]Yʻb]W\hʻbi fʻ \Rightarrow fY A Y]bi b['ØVYfʻXUgʻNi \Ui gYžʻgcbXYfbʻUi W\`ØVYfʻXYbʻ; Ugh[YVYf

6cbi g!H]dd. 'Gc``']WX 'a Y]bYa '5]fVbV!; Ugh[YVYf'Hf]b_[Y`X'[YVYb3

8]YgY: fU[Y'hUi Wkh']a a Yf'k]YXYf'Ui Zz'gY]h'k]f'X]YgYb'6Y]hfU['ni f 5]fVbV!9h]_YhhY'j YfÒZZYbh']Wkh'\UVYb"'8]Y'_i fnY'5bhk cfh'`Ui hYh. BY]bz'G]Y'gc``hYb' \Rightarrow fYa '5]fVbV!; Ugh[YVYf'_Y]b'Hf]b_[Y`X'[YVYb"

Hf]b_[Y`X`ni `[YVYb`]gh`Y]bY`A Ò[`]WX_Y]hž`K YfhgW\Àhni b[`ZØf
[i hYb`GYfj]WY`ni `nY][Ybž`UVYf`5]fVbV!; Ugh[YVYf`gY\Yb`XUgž`k Ug
g]Y`hi bž`b]W\h`U`g`GYfj]WY`Ub`g]W\` `g]Y`hY]`Yb`]\f`Ni \Ui gY
j]Y`a Y\f`a]h`X]f`U`g`; `Y]W\[YghY``hY"

 $G]Y "ibX"g]Y "_cYl]gh]YfYb"]a "gY`VYb"FUia/G]Y "X]YbYb "=\bYbb]W\h'UiZ'X]Y"[`Y]W\Y'KY]gY\Z'k]Y"Yg"XUg" < chY`dYfgcbU`'hib _\ObbhY"'8U\Yf"]gh'Yg"b]W\h'YfZcfXYf`]W\Z']\bYb"Hf]b_[Y`X'ni"[YVYb"$

'U`h1 2

 $K Ybb `G]Y `j Yfgi WXYb `a `OWXhYb ž'gY `Vgh'ni `\cghYb ž'_ObbYb `G]Y$

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\label{eq:condition} $j\ YfX]YbYb```) $ž`bUWXXYa `G]Y`=\ fY`YfghY`FY]gY`ØVYf`X]YgYb`@]b_j\ YfUbghU`hYh`\UVYb"$$ $`U`h1 5]fVbV!9h]_YhhY!H]ddg!<cgh]b[2"
```

8 Y hU]`c

6Ygi WXYb G]Y i bg Ui Z Y]bY!fY]gY "XY