

Bcfk Y[Yb'YbhXYW_Yb.: ^cfXYž'?i `hi f / Ua d/5VYbhYi Yf'ZØf'FY]gYbXY

9bhXYW_Yb^G]Y^X]Y^GW\Ob\Y]h^Bcfk Y[Ybg.^jcba U^Ygh\Ah]gW\Yb^: ^cfXYb^ØVYf^W\Ufa UbhY^Gh\AXhY^V]g^\]b^ni _i `]bUf]gW\Yb^<][\`][\hg^ibX^dfU_h]gW\Yb^FY]gYh]ddg^



Bcfk Y[YbžXYfg_UbX]bUj]gWXY; Y\Y]a h]ddžn]Y\hFY]gYbXYa]h
gY]bYf'i bVYfØ\fhYb BUhi fžZUgn]b]YfYbXYb?i `hi fYb'i bX'Y]bYf
j]Y`gWX]WXh][Yb'; YgWX]WXhY']b'gY]bYb'6Ubb"CV'Xi 'X]Y
a UYghÀh]gWXYb'; 'cfXY'Yf_i bXYbž]b'WXUfa UbhYb'GhÀXhYb
ZUb]YfYb'cXYf'gdUbbYbXY'5VYbhYi Yf']b'XYf'k]'XYb'BUhi f'Yf'YVYb
a ÒWXhYgh' Bcfk Y[Yb'V]YhYhZØf'YXYb'; YgWXa UW_XUg'F]WXh][Y"
\D'X]YgYa '5fh]_Y``bY\a Yb'k]f'X]WX'a]h'Ui Z'Y]bY'9bhXYW_i b[gfY]gY
Xi fWX'X]YgY'UhYa VYfUi VYbXY'FY[]cb'XYg'BcfXYbg"'@Ugg'i bg
[Ya Y]bgUa ']b'X]Y'i bj Yf[Ygg`]WXYb': 'cfXY'i bX'X]Y
VYY]bXfi W_YbXYb'@UbXgWXUZhYb'Y]bhUi WXYbž'X]Y'gWX ObghYb
GhÀXhY'Bcfk Y[Ybg'Yf_i bXYbž'Ui ZfY[YbXY'5_h]j]hÀhYb'ZØf
5XfYbU']bZUbg'j cfghY``Ybž'Y]bY'_i `]bUf]gWXY'; Ybi ggfY]gY'UbhfYhYbž
_i `hi fY``Y'< O\Ydi b_hY'i bX'HfUX]h]cbYb'_YbbYb`YfbYb'gck]Y

dfU_h]gWXY'H]ddg'[YVYbž'XUa]h'XY]bY'FY]gY'VY]'i bgYfYbBUWXVUfb']a 'BcfXYb'fY]Vi b[g`cg'j Yf`Ài Zh"'DUW_'XY]bY'HUgWXYbibX'aUWX'X]WX'VYfY]h'ZØf'Y]b'i bj Yf[Ygg`]WXYg'5VYbhYi Yf']bBcfkY[Yb°

I bj Yf[Ygg`]W\Y': ^cfXY'i bX UhYa VYfUi VYbXY'@UbXgW\UZhYb

Bcfk Y[Yb]gh'VY_Ubbh'ZØf'gY]bY'gdY_hU_i `ÀfYb': ^cfXYž'X]Y'g]W\k]Y'gW\]a a YfbXY'6`Ui hÒbY'nk]gW\Yb'\c\Ybž'a U'YghÀh]gW\Yb 6Yf[Yb'gW\`Ab[Y`b"'8]YgY'VYY]bXfi W_YbXYb'BUhi fd\Abca YbY g]bX'b]W\h'bi f'Y]b'<][\`][\h'ZØf'FY]gYbXYž'gcbXYfb'Ui W\'Y]b ZUgn]b]YfYbXYg'NYi [b]g'XYf'[Yc`c[]gW\Yb'; YgW\]W\hY'XYf'9fXY"8]Y': ^cfXY'YbhghUbXYb'j cf'A]``]cbYb'j cb'*J\fYb'Xi fW\'; `YhgW\YfžX]Y'g]W\']\fYb'K Y['Xi fW\'X]Y'6Yf[Y'VU\bhYbž'i bX'XUg'9f[YVb]g]gh'b]W\hg'k Yb][Yf'U`g'UhYa VYfUi VYbX"

 $9]b][Y'XYf'VY_UbbhYghYb': ^cfXY'Bcfk Y[Ybg'g]bX.$

- ; Y]fUb[YfZ^cfX.'9]b'l B9G7C!K Y`h_i `hi fYfVY'a]h K UggYfZÀ``Yb'i bX'ghY]`Yb'?`]ddYb"
- BÂfÔmZ^cfX. '9]bYf'XYf'gWNa U`ghYb': 'cfXY'XYf'K Y`hž YVYbZU``g'nia 'I B9G7C!9fVY'[Y\Òfh"
- Gc[bYZ^cfX. '8Yf'`Ab[ghY'i bX'h]YZghY': ^cfX'BcfkY[YbgžXYf'V]g'ni '%' \$, 'AYhYf'h]YZ']gh"

9]b'k Y]hYfYg'UhYa VYfUi VYbXYg'B Uhi fd\Àbca Yb'g]bX'X]Y K UggYfZÀ``Yž'X]Y'X]Y': ^cfX`UbXgW\UZh'n]YfYb"'Ni 'XYb'VY_UbbhYghYb nÀ\`Yb'X]Y'hk Y`j Y'5dcg'h`Yg'K UhYfZU``g'Ua'; Y]fUb[YfZ^cfXž'X]Y]b'XYb'Gca a Yfa cbUhYb'Y]b'VYY]bXfi W_YbXYq'GW\Ui gd]Y`'V]YhYb" #b'X]YgYf'NY]h'_Ubb'a Ub'X]Y'K UggYfZÀ``Y']b'j c``Ya :: `i gg'Yf`YVYbž
k Ybb'XUg'GW\a Y`nk UggYf'XYf'; `YhgW\Yf']b'X]Y': ^cfXY'ghfÒa h"

 $: \varnothing f'X]Y\Upsilon b][Yb\check{z}'X]Y`X]Y`?i`hi f'Bcfk Y[Ybg'Yf'YVYb'a OWWhYb\check{z}V]YhYb'j]Y`Y: \&cfXY'_`Y]bY\check{z}'WWUfa UbhY'8 OfZYf\check{z}'X]Y`a]h'HfUX]h]cbYbibX'; YgWW]WWhYb'Ui Zk UfhYb''CfhY'k]Y'jiYgibX'ibX': `Ua 'g]bXb]WWh'bi f'Z\notingf']\fY'a U`Yf]gWWY'GWWOb\Y]h'VY_Ubbhž'gcbXYfb'Ui WWZ\notingf']\fY'_i`]bUf]gWWYb''GdYn]U`]hÀhYb'ibX'ZfYibX`]WWYb9]b\Y]a]gWWYb'''<]Yf'_Ubb'a Ub'X]Y'bcfk Y[]gWWY; UghZfYibXgWWUZhYf`YVYb'ibX'X]Y'_Ogh`]WWY''c_U`Y'?\notingWY dfcV]YfYb''$

B UhØf`]W\]gh'Y]b'6Ygi W\]b'Bcfk Y[Yb'b]W\ h'j c``ghÀbX][žc\bY'X]Y
A Ò[`]W_Y]hž'X]Y'BcfX`]W\ hYf'ni 'VYcVUW\ hYb'''6YgcbXYfg']b'XYb
: 'cfXfY[]cbYb']a 'K]bhYf'g]bX'X]Y'5i ZbU\a Yb'XYg'5i fcfU'6cfYU`]g
Y]bZUW\ i bj Yf[Ygg`]W\ "'8]Y'hUbnYbXYb'@]W\ hYf'Ua '<]a a Y`
gd]Y[Y`b'g]W\ ']b'XYb'fi\][Yb'K UggYfcVYfZ`AW\Yb'XYf': 'cfXY'k]XYf
i bX'gW\UZZYb'Y]b'a U[]gW\Yg'GW\Ui gd]Y`ž'XUg'X]Y'<YfnYb'XYf
FY]gYbXYb'YfcVYfh"

: ^cfX	<][\`][\ hg	I B 9G7C!
		K Y`h i `hi fYfVY

; Y]fUb[YfZ'cfX	K UggYfZÀ``Yž	¥
	DUbcfUa UV`]W <u>Y</u>	
BÂ fÔmZ°cfX	GW a U`Y	从
	9]bgWXb]hhYž	
	XfUa Uh]gW\Y	
	@UbXgWUd	
Gc[bYZ'cfX	@Àb[ghYf': ^cfXž	BY]b
	j]Y`nÀ\`][Yʻ≢bgY`b	

K Ybb'G]Y'Bcfk Y[Yb'VYgi WYbž'Vf]b[Yb'G]Y'→fY'?Ua YfU'i bX
Y]b][Y'[i hY'K UbXYfgWi\Ya]hž'XYbb'X]Y'5i gg]W\h'k]fX'G]Y
Y]bZUW\ia\UiYb"'CV'G]Y'X]Y'a U'YghÀh]gW\Yb': 'cfXY'Yf_i bXYbž
Y]bYb'K UggYfZU``'VYk i bXYfb'cXYf'X]Y'ZUgn]b]YfYbXYb'6Yf[Y
VYghY][Ybž'Bcfk Y[Yb'V]YhYh'i bj Yf[Ygg`]W\Y'9f`YVb]ggYž'X]Y'bcW\
`Ub[Y'bUW\'→fYf'FØW__Y\f']a'; YXÀW\hb]g'V`Y]VYb'k YfXYb"': Øf
k Y]hYfY' →bZcfa Uh]cbYb'ØVYf'X]Y'UhYa VYfUi VYbXY'@UbXgW\UZh
Bcfk Y[Ybg'VYgi W\Yb'G]Y'X]Y'cZZ]n]Y``Y'K YVg]hY'j cb'J]g]h
Bcfk Um"

8]Y W Ufa UbhYghYb GhÀXhY Bcfk Y[Ybg

 $Bcfk\ Y[\ Yb\]gh'b]Wk\ h'bi\ f'\ ZØf'gY]bY'VYY]bXfi\ W_YbXYb':\ ccfXY'ibX \\ gdY_hU_i\ `AfYb'BUhi\ f`UbXgWkUZhYb'VY_Ubbhž'gcbXYfb'Ui Wk'\ ZØf \\ gY]bY'WkUfa\ UbhYb'GhAXhYž'X]Y'Y]b'Y]bn][\ Ufh][\ Yg':\ `U]f'ibX \\ _i\ `hi\ fY``Y'9f`YVb]ggY'V]YhYb'''<]Yf'g]bX'Y]b][\ Y'XYf'\ ZUgn]b]YfYbXghYb \\ GhAXhYž'X]Y'G]Y'k\ A\fYbX'=\ fYf'FY]gY'Xi\ fWk'Bcfk\ Y[\ Yb'i\ bVYX]b[\ h\ VYgi\ WkYb'gc``hYb'' \\$

$Cg^c.8]Ydi^g]YfYbXY < UidhghUXh$

Cg`cž'X]Y`<Ui dhghUXh'Bcfk Y[Ybgž']gh'Y]bY'a cXYfbY'A Yhfcdc`Yž X]Y'HfUX]h]cb'i bX'\dbcj Uh]cb'dYfZY_h'_ca V]b]Yfh"'Jca VYY]bXfi W_YbXYb'CdYfb\Ui g'V]g'ni a '\]ghcf]gW\Yb : Yghi b[gVYfY]W\'5_Yfg\i g'[]Vh'Yg'\]Yf'j]Y`Y'GY\Ybgk ØfX][_Y]hYb ni 'YbhXYW_Yb"'8]Y'GhUXh']gh'Ui W\'VY_Ubbh'ZØf.

- AigYYb. '8 Ug'K]_]b[YfgWX]ZZaigYia'ibX'XUg'AibWX! AigYia'g]bX'bif'Y]b][Y'XYf'_i`hifY``Yb'<][\`][\hg"
- DUf_g. '8 Yf'J][Y`UbX!DUf_ ']gh'VYfØ\a h'ZØf'gY]bY
 Y]bn][Ufh][Yb'G_i `dhi fYb'i bX'gW\ObYb'; fØbZ`ÀW\Yb"
- FYghUi fUb hg. 'Cg`c'V]YhYh'Y]bY'`YVYbX][Y'FYghUi fUb h! GnYbY'a]h'GW\kYfdi b_h'Ui Z'bUW\\U`h][Yf'i bX'`c_U`Yf ? ØW\Y"

6Yf[Yb.8]YGhUXhXYfg]YVYb6Yf[Y

 $6Yf[\ Ybz`X]Y`nk\ Y]h[\ f\grave{O} \rangle hY`GhUXh`Bcfk\ Y[\ Ybgz`k\]fX`cZh`U`g`XUg`Hcf`ni$

- 6fm[[Yb.8]YgYg'l B9G7C!K Y`h_i `hi fYfVY`nY][h'X]Y'U`hYb <c`n\Ài gYf'XYf'<UbgYUh]gW\Yb'<UbXY`gnY]h"
- : `ÔmY b. '9]bY': U\fh'a]h'XYf': `Ô]VU\b'Ui Z'XYb': `ÔmYb V]YhYh'UhYa VYfUi VYbXY'5i gV`]W_Y'Ui Z'X]Y'GhUXh'i bX'X]Y I a [YVi b["
- : $]gWX a Uf_h. DfcV]YfYbG]YZf]gWXYA YYfYgZfØWXhYi bX YfZU\fYbG]Ya Y\f\ØVYfX]Y``c_U`Y:]gWXYfY]"$

GhUj Ub [Yf. ?i `]b Uf]gW $XY < O Ydib_hY ib X ?i `hif$

8]Y'GhUXh'GhUj Ub[Yfž'X]Y'U`g'²`!<Ui dhghUXh'BcfkY[Ybg 'VY_Ubbh]ghž'\Uh'Ui W\'Y]bY'W\Ufa UbhY'5`hghUXh'a]h'kY]»Yb'<c`n\Ài gYfbibX'[Ya Øh`]W\Yb'; UgqYb"'<]Yf'Z]bXYb'G]Y.

- ?i bgh[U`Yf]Yb. `GhUj Ub[Yf`\Uh'Y]bY``YVYbX][Y
 ?i bghgnYbYž'X]Y`]b`nU\`fY]WXYb'; U`Yf]Yb'i bX'5hY`]Yfg dfÀgYbh]Yfh'k]fX"
- 8 Ug B c fk Y[]gW Y 9fX O`a i gYi a . *\(\phi\)Zcfa Uh]j 'i bX i bhYf\U`hgUa ž'V]YhYh'Yg'Y]bYb 'h]YZYb'9]bV`]W_']b'X]Y
 YgW [W hY XYf'2 `]bXi ghf]Y"
- ; Ughfcbca]Y. '8]Y 'GhUXh']gh'VY_Ubbh'ZØf']\fY '_fYUh]j Yb F YghUi fUbhgž'X]Y 'Zf]gW\Yž'`c_U`Y 'Ni hUhYb'j Yfk YbXYb"

HfcbX\Y]a . '9]bY'GhUXh'XYf'; YgWX]WX hY'i bX 6]`Xi b[

 $\label{thm:linear_equation} HfcbX\Y]a\ \check{z}'Y]bY'XYf'\grave{A}`hYghYb'Gh\grave{A}XhY'Bcfk Y[Ybg\check{z}']gh'XUg'Ni \Ui gYXYf'B]XUfcg_Uh\YXfU`Y\check{z}'XYf'[f\grave{O}>hYb'?]fWXY'XYg'@UbXYg'''G]Y']gh'Y]bk]WXh][Yf'D]`[Yfcfh'i bX'Y]b'AY]ghYfkYf_'XYf'[ch]gWXYb5fWX]hY_hi f'''\delta''HfcbX\Y]a'_\ObbYb'G]Y'Ui WX.$

- 8 Ug '9fnV]gW c Zgd U`U]g. '9]b '\]ghcf]gW\Yg'; YVÀi XYž'XUg 9]bV`]W_Y']b 'X]Y'V]gW\ÒZ]W\Y'; YgW\]W\hY'V]YhYh"
- 8]Y Ghi XYbhYbghUXh. 8]Y HYWXb]gWX!
 BUhi fk]ggYbgWXUZh`]WXY I b]j Yfg]hÀh'Bcfk Y[Ybg'fBHBIŁ

- jYfY]\hXYfGhUXhY]b 1b[YgibXXmbUa]gWXYg5aV]YbhY"
- 9]bY``YVYbX][Y'?i`hifgnYbY.`AigYYbž': Ygh]j U`g'ibX Y]b'fY[Yg'BUW\h`YVYb'qcf[Yb'ZØf'5VkYW\g`ib["

i Ygi bX. 9]bY GhUXh]a \Rightarrow [YbXgh]

¡`Ygi bX`]gh'VY_Ubbh'ZØf`]\fY`VYY]bXfi W_YbXY`5fWk]hY_hi f`]a ⇒ [YbXgh]``i bX``]Y[h'hfUi a \UZh'Ub'XYf'K Ygh_ØghY"'9[U`ž'cV'G]Y'X]Y GhUXh'ni ': i » 'Yf_i bXYb'cXYf'X]Y`l a [YVi b['[Yb]Y»Yb'a ÒWkhYbž ¡`Ygi bX`\Uh'j]Y``ni 'V]YhYb.

- > i [YbXgh]`! < Ai gYf. '9bhXYW_Yb'G]Y'X]Y'Y]bn][Ufh][Yb
 ; YVAi XYž'X]Y'bUWX'XYa 'GhUXhVfUbX'j cb'%-\$('YfVUi h
 k i fXYb"
- 8]Y '5i gg]Wk hgd `UhhZcfa '5_g`U. 'Jcb'\]Yf'Ui g'\UVYb'G]Y
 Y]bYb'ZUbhUgh]gWkYb'6`]W_'ØVYf'X]Y'GhUXh'i bX'X]Y
 i a `]Y[YbXY'BUhi f"
- A Uf]h]a Y'; YgWX]WX hY. '8 Ug' *> [YbXgh]`a i gYi a '[]Vh
 \$\text{bYb'9}\text{bV`}]\text{W_Y'}\text{b'X}]Y'; YgWX]WX hY'XYg'GYYj Yf_Y\fg']b'XYf
 FY[]cb"

Cg`c!6YfY]WX i bX 6YmcbX. 8]Y _`Y]bYb WX Ufa UbhYb GhÀXhY

5V[YgY\Yb'] cb'XYb'[fÒ» YfYb'GhÀXhYb'\Uh'Bcfk Y[Yb'nU\`fY]WXY
_`Y]bYfY'WXUfa UbhY'GhÀXhY'i bX'CfhYž'X]Y'Y]bYb'6Ygi WX'k Yfh'g]bX.

- 8fUa a Yb. A]h'XYa :: `i gg'8fUa a YbgY`j U'i bX'gW\ÒbYb DUf_qž']XYU``ZØf'Ci hXccf!5_h]j]hÀhYb"
- @]``Y\Ua a Yf. '6Y_UbbhY'K]bhYfghUXh'a]h'XYf
 C`ma d]UfYbU'i bX'\ØVgW\Yb'<c`n\Ài gYfb"
- Fcfcg. '9]b'l B9G7C!K Y`h_i `hi fYfVYž'XUg'ZØf'gY]bY'[i h Yf\U`hYbYb'6Yf[k Yf_gUfWX]hY_hi f'VYfØ\a h']gh"

XXY'X]YgYf'GhÀXhY'\Uh']\fYb'Y][YbYb'7\UfU_hYf'i bX'7\Ufa Y'i bX
hfÀ[h'ni 'XYa 'j]Y`ZÀ`h][Yb'9f`YVb]g'VY]ž'XUg'Bcfk Y[Yb'ni 'V]YhYb
\Uh'''9[U`ž'cV'G]Y'X]Y'; YgWX]WXhYž'X]Y'?i `hi f'cXYf'X]Y
_i `]bUf]gWXYb'; YbØggY'gi WXYbž'Bcfk Y[Ybg'GhÀXhY'\UVYb'ZØf

5 V Y b h Y i Y f 'i b X '5 _ h] j] h À h Y b '] b 'X Y f 'B U h i f

Bcfk Y[Yb']gh'Y]b'k U\fYg'DUfUX]Yg'ZØf'BUhi f`]YV\UVYf'i bX $5VYbhYi Yfgi WkYbXY'''Jca 'a U'YghÀh]gWkYb': 'cfX'V]g'\]b'ni 'XYb gWkbYYVYXYW_hYb';]dZY`b']b'XYb'; YV]f[Yb'''X]Y'bUhØf`]WkY GWk Òb\Y]h'XYg'@UbXYg'`ÀXh'ni 'Y]bYf'J]Y`nU\`'j cb'5_h]j]hÀhYb']a : fY]Yb'Y]b'''<]Yf'g]bX'Y]b][Y'XYf'Ui ZfY[YbXghYb'5VYbhYi Yfž'X]Y'G]Y]b'Bcfk Y[Ybg'i bVYfØ\fhYf'BUhi f'Yf`YVYb'_ÒbbYb.$

- K UbXYfi b[Yb]b'UhYa VYfUi VYbXYb'@UbXgW UZhYb.

 Bcfk Y[Yb'V]YhYh'i bnÀ\`][Y'K UbXYfk Y[Yž'X]Y'Xi fW\
 gdY_hU_i `ÀfY'@UbXgW\UZhYb'ZØ\fYb"'6Y`]YVhY'K UbXYfn]Y`Y
 g]bX'XYf'DfY]_Yghc`Yb'fDfYX][hghi \`Ł'i bX'XYf'Hfc``hi b[U
 fHfc``ni b[YŁž'X]Y'ZØf']\fY'VYY]bXfi W_YbXYb'5i gg]W\hYb
 VY_Ubbh'g]bX"
- : ^cfX!Hci fYb. '9f_i bXYb'G]Y'X]Y'VYfØ\a hYb': ^cfXY
 Bcfk Y[Ybgž'k]Y'XYb'; Y]fUb[YfZ^cfX'i bX'XYb'BÂ fÔmZ^cfXž
 Ybhk YXYf'dYf'6cch'cXYf'?UmU_"'8]YgY'Hci fYb'V]YhYb'Y]bY
 Y]ba U`][Y'DYfgdY_h]j Y'Ui Z'X]Y'ghY]`Yb'?`]ddYb'i bX
 K UggYfZÀ``Y"
- G_]ZU\fYb'i bX'Gbck VcUfXYb. Bcfk Y[Yb']gh'Ui W\'Y]b
 VY`]YVhYg'N]Y`'ZØf'K]bhYfgdcfh`Yf"'FY[]cbYb'k]Y'< Ya gYXU`
 i bX'Hfmg]`'V]YhYb'nU\`fY]W\Y'D]ghYb'i bX'g]bX'gck c\``ZØf
 5bZÀb[Yf'U`g'Ui W\'ZØf': cfh[YgW\f]hhYbY'[YY][bYh"
- BcfX`]WX hYf`VYcVUWX hYb. '=b`XYb`K]bhYfa cbUhYb
 j Yfk UbXY`h`g]WX 'Bcfk Y[Yb`]b`Y]b`a U[]gWX Yg`@]WX hgd]Y`ž
 k Ybb`X]Y`5i fcfU`6cfYU`]g`XYb`<]a a Y``Yf`Yi WX hYh" '8]Y
 VYghY`NY]hži a 'X]Y`BcfX`]WX hYf`ni 'gY\Ybž']gh`nk]gWXYb
 C hcVYf`i bX`A Àfn"
- 5b[Y`b'i bX'6cchgZU\fhYb.'8]Y'K UggYfk Y[Y Bcfk Y[Ybg'g]bX']XYU`ni a '5b[Y`b"'CV']a 'GØ» k UggYf cXYf']a 'A YYfž'XUg'5b[Y`b']b'Bcfk Y[Yb'n]Y\h'gck c\` 5bZÀb[Yf'U`g'Ui W\'DfcZ]g'Ub"

: $\emptyset f'X]Y'a i h][Yb'5VYbhYi fYf'[]Vh'Yg'Ui W\'X]Y'A Ò[`]W_Y]hž'VY]a 'FUZh]b['Ui Z'XYb'k]`XYb': `<math>\emptyset ggYb'BcfkY[Ybgž'k]Y'XYa''G^cU'cXYf$

XYa 'Chfuž'a]hni a UWXYb"'<]Yf'_ÒbbYb'G]Y'a]h'XYf Ghfca gWXbY``Yb'\YfUi g[YZcfXYfh'k YfXYb'i bX'[`Y]WXnY]h]['X]Y UhYa VYfUi VYbXY'@UbXgWXUZh'[Yb]Y»Yb"'8]YgY'5XfYbU`]bgWXØVY g]bX'Y]bY'XYf'VYghYb'A Ò[`]WX_Y]hYbž'X]Y'GWXÒb\Y]h'XYf bcfk Y[]gWXYb'BUhi f'\Ui hbU\'ni 'Yf`YVYb"

? `YhhYfb i bX 6Yf[ghY][Yb

 $8] Y \cdot ghY] \dot{Y}b :: Y \dot{g}Yb \dot{I} b X \dot{f};] d ZY \dot{B} c fk Y [Ybg \dot{I}n] Y \dot{Y}b \dot{U} i W \dot{J}] Y \dot{Y} \\ ? \dot{Y}hhYfYf \dot{I} b X \dot{G}Yf [ghY] [Yf \dot{U}b \ddot{B}] Y \cdot @c Zc hY b ! <math>\pm gY \dot{b} \dot{V}$] Y hY b \dot{B}] W h \dot{B} i f UhYa VYf Ui VYb XY \dot{B} i gV \dangle W_Y \dot{Z} g c b XYfb \dot{U} i W \dot{Y}f Ui g Zc f XYfb XY F c i hY b \dangle Z0 f ? \dangle YhhYf VY [Y]ghYfhY \dangle S Yf VYf \Q \a hY \dangle GhYh] b X \dangle [] \dot{h} \dot{U} \dot{g} Y] b Yf \dot{X}Yf \dot{g}W k] Yf [ghYb';] d ZY \dot{B} c fk Y [Ybg'i b X']ghY] b A i gg' Z0 f Yf ZU \dangle f YbY' \dangle GYf [ghY] [Yf"

FUXZU\fYb Xi fW Bcfk Y[Ybg BUhi f

: Øf'FUX`]YV\UVYf']gh'Bcfk Y[Yb'YVYbZU``g'Y]b'HfUi a n]Y`"'8Ug @UbX'V]YhYh'j]Y`Y'FUXk Y[Yž'X]Y'Xi fW\'a U`Yf]gW\Y'8ÒfZYfž Xca]b]YfYbXY': ^cfXY'i bX'a U^YghÀh]gW\Y'6Yf[Y'ZØ\fYb"'8]Y'FU``UfjY[Ybž'Y]bY'XYf'VY_UbbhYghYb'FUXghfYW_Ybž'ZØ\fh'Xi fW\gdY_hU_i `ÀfY'@UbXgW\UZhYb'i bX'\Uh'\]ghcf]gW\Y'6YXYi hi b["

K]`Xh]YfVYcVUW\hib[Yb

9]b'k Y]hYfYg'i bj Yf[Ygg`]W\Yg'5VYbhYi Yf']gh'X]Y'6YcVUW\hi b['j cb K]`Xh]YfYb"'Bcfk Y[Yb']gh'X]Y'< Y]a Uh'Y]bYf'J]Y`nU\`'j cb'H]YfUfhYbž XUfi bhYf'F Ybh]YfYž'9`W\Y'i bX'K \U`Yg"'6YgcbXYfg'VY`]YVh'g]bX K U`VYcVUW\hi b[ghci fYb']b'XYb'; Yk ÀggYfb'fi bX'i a 'Hfca gÔ'i bX 5bXYbYgž'k c'G]Y'X]Y'; Y`Y[Yb\Y]h'\UVYbž'j YfgW\]YXYbY'K U`UfhYb ni 'gY\Ybž'XUfi bhYf'CfWJq'i bX'6i W_Y`k U`Y"

VYfbUWX hi b[g! i bX I bhYfVf]b[i b[ga Ò[`]WX_Y]hYb

8]Y'9f`YVb]ggY']b'XYf'BUhi f'_ÒbbYb'bcW\'i bj Yf[Ygg`]W\Yf
[Ya UW\h'k YfXYbž'k Ybb'G]Y'g]Y'a]h'XYf'f]W\h][Yb'I bhYf_i bZh
_ca V]b]YfYb"'=b'Bcfk Y[Yb'\UVYb'G]Y'X]Y'A Ò[`]W_Y]hž']b

[Ya Øh`]WXYb`< ØhhYbž'hfUX]h]cbY``Yb`:]gWXYf\Ài gYfb`fFcfVi YfŁcXYf`]b`acXYfbYb`<chY`g`ni `ØVYfbUWXhYb": Øf`X]Y5VYbhYi Yf`i gh][Yb`[]Vh`Yg`Ui WX`X]Y`AÒ[`]WX_Y]hž']a`NY`h`cXYfVY]a`K]`XWUad]b[`ni `ØVYfbUWXhYbž'k Ug`]b`Bcfk Y[Yb``Y[U``i bXkY]h'j YfVfY]hYh']gh"

5_h]j]hÀh	9a dZc\`YbY`FY[]cb
K UbXYfb	DfY]_Yghc`Ybž'Hfc``hi b[U
: ^cfX!Hci fYb	; Y]fUb[YfZ^cfXžBÂfÔmZ^cfX
$G_]ZU \setminus fYb$	< Ya gYXU`ž'Hfmg]`
BcfX`]W\hYf'VYcVUW\hYb	BcfXbcfk Y[Yb
FUZh]b[G^cUž'ChfU
?`YhhYfb	@c Zc hY b
KU`VYcVUW\hib[Hfca gÔž 5bXYbYg

 $8]Y'J]Y`ZU`h`XYf`5VYbhYiYf`ibX`5_h]j]hÀhYb`]b`BcfkY[Ybg`BUhifV]YhYh`ZØf`^YXYb`YhkUg"`9[U`ž`cV`G]Y`X]Y`Fi\Y`XYf`BUhif`giW\YbcXYf`bUW\`XYa`bÀW\ghYb`[fc\)Yb`5XfYbU`]bgW\iV`ghfYVYb`BcfkY[Yb`k]fX`G]Y`a]h`gY]bYb`UhYaVYfUiVYbXYb`@UbXgW\UZhYbibX`UiZfY[YbXYb`AÒ[`]W_Y]hYb`ZØf`5VYbhYiYf`VY[Y]ghYfb"$

?i`]bUf]gW\Y`9bhXYW_i b[gfY]gY`Xi fW\Bcfk Y[Yb

8]Y'bcfk Y[]gWXY'?ØWXY']gh'Y]b'gWX a UW_\UZhYg'Ni gUa a Ybgd]Y`Ui g'HfUX]h]cbžfY[]cbU`Yb'Ni hUhYb'i bX'_fYUh]j Yb'9]bZØggYb"
K Ybb'Xi 'Bcfk Y[Yb'VYgi WXghž'k]fgh'Xi 'b]WXh'bi f'j cb'XYf
UhYa VYfUi VYbXYb'BUhi f'VY[Y]ghYfh'gY]bž'gcbXYfb'Ui WX'j cb'XYb
_Ògh`]WXYb'; Yf]WXhYbž'X]Y'XUg'@UbX'ni 'V]YhYb'\Uh"'8]Y
_i`]bUf]gWXY'@UbXgWXUZh'Bcfk Y[Ybg'k]fX'[YdfÀ[h'j cb'Zf]gWXYb
A YYfYgZfØWXhYbž'\Yfn\UZhYb': `Y]gWX[Yf]WXhYb'i bX'Y]bYf'J]Y`nU\`j cb'6YYfYb'i bX'; Ya ØgY"

9]b 'UVgc`i hYg'A i gg'ZØf': Y]bgW\a YW_Yf']gh'XYf'; Ybi gg'j cb
Zf]gW\Ya '@UW\g''Bcfk Y[Yb']gh'VYfØ\a h'ZØf'gY]bY'@UW\gni W\h'i bX
XYf'; Ybi gg'j cb'[YfÀi W\YfhYa 'cXYf'[Y[f]``hYa '@UW\g']gh'Y]bY
k U\fY'8Y`]_UhYggY'''J]Y`Y'FYghUi fUbhg'V]YhYb'@UW\g[Yf]W\hY'Ui g
bUW\\U`h][Yf'Ni W\h'Ubž'i bX'Y]b'6Ygi W\']b'Y]bYa ':]gW\a Uf_h'k]Y

XYa :]gWX a Uf_h']b'6Yf[Yb']gh'Y]b'9f`YVb]g'ZØf'g]WX"'<]Yf'_Ubbgh Xi 'X]Y'J]Y`ZU`h'XYf'A YYfYgZfØWXhY'dfcV]YfYbž'XUfi bhYf'? fYVgž' GYY[i f_Yb'i bX'A i gWXY`b"

Bcfk Y[Yb']gh'ni XYa 'VY_Ubbh'ZØf'gY]bY'fY[]cbU`Yb

? ÀgYgcfhYb"'8Yf'VY_UbbhYghY'? ÀgY']gh'6fi bcghž'Y]b'VfUi bYf

N]Y[Yb_ÀgYž'XYf'gØ»`]W\'i bX'_UfUa Y``Ufh][']a '; YgW\a UW_']gh''9f

k]fX'hfUX]h]cbY``'Ui Z'6fch'cXYf'K UZZY`b'gYfj]Yfh'i bX']gh'Y]bY

VY`]YVhY'9f[Àbni b['VY]a ': fØ\ghØW_'cXYf'U`g'GbUW_"': Øf

? ÀgY`]YV\UVYf'[]Vh'Yg'gc[Uf'? ÀgYhci fYbž'VY]'XYbYb'Xi
j YfgW\]YXYbY'? ÀgYgcfhYb'i bX'XYfYb'< YfghY``i b['_YbbYb`YfbYb
_Ubbgh"

8]Y'bcfk Y[]gWXY'BUhi f'ghY``h'Ui WX'j]Y`Y'_Ògh`]WXY'Ni hUhYb'VYfY]hž X]Y']b'XYf'?ØWXY'JYfk YbXi b['Z]bXYb"'K U`XVYYfYbž'k]Y'n"6"' 6fca VYYfYbž'<Y]XY`VYYfYb'i bX'DfY]gY`VYYfYbž'g]bX VYgcbXYfg'VY`]YVh'i bX'k YfXYb'cZh']b'8YggYfhgž'A Ufa Y`UXYb'cXYf U`g'6Y]`U[Y'ni ': `Y]gWX[Yf]WXhYb'j Yfk YbXYh"'8]Y'9fbhYnY]h']a Gca a Yf']gh'Y]b'YWXhYf'<cWX[Ybi gg'ZØf'BUhi f`]YV\UVYf"

 $: \emptyset f `Y] b `i b j `Y f [Y g g `] W `Y g `_i `] b U f] g W `Y g `9 f `Y V b] g `Y a \ d Z Y \setminus `Y `] W ` \check{z} `U b Y] b Y f `b c f k `Y [] g W `Y b `? c W _ `U g g Y `h Y] `ni b Y \ a `Y b " `\delta 'j] Y `Y b$

GhÀXhYb'_Ubbgh'Xi '`YfbYbž'k]Y'a Ub'hfUX]h]cbY``Y'; Yf]W\hY
ni VYfY]hYhž'i bX'X]Y'; Y\Y]a b]ggY'XYf'bcfkY[]gW\Yb'?ØW\Y
YbhXYW_Yb"'C Zha U`g'k YfXYb'X]YgY'?i fgY'j cb'@cWU`g'[Y`Y]hYhž'X]Y
]\fY'@Y]XYbgW\UZh'ZØf'XUg'?cW\Yb'a]h'X]f'hY]`Yb"

6Y`]YVhY; Yf]WXhY']b'BcfkY[Yb

; Yf]W\h	6YgWkfY]Vib[
@UWX g	: f]gW\`cXYf`[YfÀiW\Yfhž'dYfZY_h
	ZØf 'YXYg'; Yf]WXh"
?`]ddZ]g_	; YhfcW_bYhYf'i bX'[YgU`nYbYf
	?UVY`′Ui ž'\Yfn\UZh'i bX
	hfUX]h]cbY``"
: Áf]_Á`	@Ua a ZY]gWX i bX ?? c \ ``
	Bcfk Y[YbgBUh]cbU`[Yf]WXh"
6fi bcgh	GØ» Yf'N]Y[Yb_ÀgYž']XYU``ZØf'X]Y
	: fØ\ghØV <u>g</u> hUZY`"
K U`XVYYfYb	: f]gW\Y`6YYfYb`Ui g`XYb
	K À`XYfbž'Y]b'YWXhYg'<][\`][\h
	XYf'BUhi f"

9]b'k Y]hYfYg'<][\`][\h'ZØf'@]YV\UVYf'XYf'; Ughfcbca]Y'g]bX'X]Y
nU\`fY]W\Yb': ccX': Ygh]j U`gž'X]Y'k À\fYbX'XYg'[UbnYb'*\U\fYg']b
Bcfk Y[Yb'ghUhhZ]bXYb'''<]Yf'_Ubbgh'Xi 'b]W\h'bi f'X]Y'VYghYb
`c_U`Yb'DfcXi _hY'dfcV]YfYbž'gcbXYfb'Ui W\'Ub'K cf_g\cdg'i bX
J Yf_cghi b[Yb'hY]`bY\a Yb'''9]bYg'XYf'VY_UbbhYghYb': Ygh]j U`g']gh
XUg'6Yf[Yb'A UhZYgh]j U`ž'XUg']a '5i [i gh'[YZY]Yfh'k]fX'i bX
_i`]bUf]gW\Y'?Ògh`]W_Y]hYb'Ui g'XYf'FY[]cb']b'XYb'A]hhY`di b_h
ghY``h''

 $\label{eq:converse_converse_converse} $$JYf[]gg'b]WhizUiWXX]YbcfkY[]gWXYGWXc_c`UXYibX`\\ 6UW_kUfYb'ni'dfcV]YfYb''8]Y'bcfkY[]gWXY\\ GWXc_c`UXYb\YfghY``ib['\Uh'g]WX']b'XYb'`YhnhYb'\\J\fYb'ghUf_kY]hYfYbhk]W_Y`hz'ibX'Xi'Z]bXYgh'j]Y`Y'_`Y]bY'7\cWc`Uh]Yfgz'X]YUi <math>_Yf[YkO)$ b']WXY'?fYUh]cbYb'UbV]YhYb'''

8]Y'?ca V]bUh]cb'Ui g'fY[]cbU`Ybž'bUW\\U`h][Yb'Ni hUhYb'i bXhfUX]h]cbY``Yb'FYnYdhYb'gcf[h'XUZØfž'XUgg'X]Y'bcfk Y[]gW\Y

; Ughfcbca]Y 'Y]b 'k U\f\UZh'Y]bn][Ufh][Yg '9f`YVb]g 'V]YhYh'' '5`gcžgWX Ui ']b 'X]Y 'GdY]gY_UfhYbž'gdf]WX 'a]h 'XYb '9]b\Y]a]gWXYb 'ØVYf]\fY '9a dZY\`i b[Yb 'i bX ``Ugg 'X]WX 'Ui Z'Y]bY '_i `]bUf]gWXY 9bhXYW_i b[gfY]gY 'Y]bž'X]Y 'Xi 'gc 'gWX bY```b]WX h'j Yf[YggYb 'k]fgh'

?i `hi fY``Y <][\ `][\ hg i b X `HfUX]h]c b Y b

Bcfk Y[Yb']gh'Y]b'@UbXžXUg'fY]W\'Ub'_i `hi fY``Yb'<][\`][\hg'i bXh]YZ'j Yfk i fnY`hYb'HfUX]h]cbYb']gh'''Jcb'XYb'ZUfVYbZfc\Yb

<c`n\Ài gYfb']b'XYb'GhÀXhYb'V]g'\]b'ni 'XYb'U'hYb'6fÀi W\Ybž'X]Y

j cb'; YbYfUh]cb'ni '; YbYfUh]cb'k Y]hYf[Y[YVYb'k YfXYbž'V]YhYh

Bcfk Y[Yb'Y]bY'ZUgn]b]YfYbXY'A]gW\ib['Ui g'JYf[Ub[Yb\Y]h'i bX

; Y[Ybk Ufh''8]Y'?i `hi f'XYf'Bcfk Y[Yf']gh'Yb['a]h']\fYf

BUhi fj YfVi bXYb\Y]h'i bX']\fYa '@YVYbggh]`'j Yf_bØdZhž'k Ug'g]Y']a

5``hU['i bX']b': Y]Yf`]W_Y]hYb'ni a '5i gXfi W_'Vf]b[Yb''

HfUX]h]cbY``Y: YghYibX: Y]YfhU[Y

- 8 UgʻA]hhgca a YfZYghʻfGUb_h\UbgŁ. 5a &' "'\dagga b]`k]fX XUgʻA]hhgca a YfZYghʻa]hʻ: Yi YfbžʻA i g]_ʻi bXʻHUbnʻ[YZY]Yfh" 8]Ygʻ]ghʻY]bYʻ; YʻY[Yb\Y]hžʻi a 'X]Yʻ`Ab[ghYbʻHU[YʻXYg \lambda\fYgʻni '[Yb]Y» Yb"
- 8 Ug B Uh]cbU`ZY]YfhU['ff%+"'A U]Ł. '8 Yf GmhhYbXY'A U]']gh XYf bUh]cbU`Y': Y]YfhU['Bcfk Y[Ybgž'Ub'XYa 'X]Y I bUV\Àb[][_Y]h'j ca 'gWk YX]gWXYb'? Ob][fY]WX'[YZY]Yfh k]fX"'DUfUXYb'i bX'ZYgh`]WX'[Y_`Y]XYhY'A YbgWXYb'dfÀ[YbX]Y'GhfU» Yb'XYf GhÀXhY"
- 8 Ug K Y]\ b UW hYb flai `Ł K Y]\ b UW hYb `]gh Y]b Y XYf k]W h][gh Yb ': Y]Yf `]W _Y]h Yb `]b `B c fk Y[Yb ž `[Yd f A [h 'j c b H f U X]h]c b Yb `k]Y `XYa `5 X j Yb hg _U `Yb XY f ž `XYa K Y]\ b UW hg V Ui a `i b X `XYa `: Ygh Ygg Yb `Ua `< Y]`][UV Yb X "

< c WX nY]hghfUX]h]c b Y b</pre>

Bcfk Y[]gWXY'<cWxnY]hYb'g]bX'\Ài Z]['Y]bY'gWX ÒbY'?ca V]bUh]cb Ui g'U`hYb'6fÀi WXYb'i bX'a cXYfbYb'9`Ya YbhYb"'9]b'hfUX]h]cbY``Yf <cWxnY]hgi a ni ['_Ubb'j cb'A i g]_'i bX'j c`_ghØa `]WXYf'HfUWXh VY[`Y]hYh'k YfXYb"'6fUi hdUUfY'k À\`Yb'cZh'HfUWXhYb'Ui g']\fYb FY[]cbYbž'X]Y'X]Y'_i `hi fY``Y'<Yf_i bZh'ni f'GWXUi 'ghY``Yb"

$6Y^{Y} V h Y < c W n Y hghf U X h c b Y b$.

- 8 Ug' < YVYb 'XYg'GW 'Y]Yfg. '8 Yf'6fUi h'k]fX'c Zh'XYf
 GWX 'Y]Yf'j cb']\fYa 'J UhYf'cXYf'Y]bYa 'UbXYfYb
 : Ua]`]Yba]h[`]YX'UV[Ybca a Ybži a 'ni 'gma Vc`]g]YfYbž
 XUgg'g]Y'bi b'cZZ]n]Y``ni f': Ua]`]Y'XYg'6fÀi h][Ua g'[Y\Ôfh"
- ?i bghj c``Y'HcfhYb.'9]bY'a Y\fghÒW_][Y'<cW\nY]hghcfhY gd]Y`h'Y]bY'nYbhfU`Y'Fc``Y'i bX'k]fX'cZh'_i bghj c```XY_cf]Yfh"
- Dc`hYfUVYbX'fDc`hYfUVYbXŁ.'6Yj cf'XUg'9\YdUUf \Y]fUhYhž'Z]bXYh'Y]b'Dc`hYfUVYbX'ghUhhž'VY]'XYa ': fYi bXY i bX': Ua]`]Y'ni gUa a Yb_ca a Ybž'i a 'ni 'ZY]Yfb'i bX dYfgÒb`]WXY'; YgWXYb_Y'ni 'ØVYffY]WXYb"

: c`_cfY i bX A mh\Yb

Bcfk Y[Yb']gh'fY]W\"Ub': c`_`cfY'i bX'U`hYb'A mh\Ybž'X]Y'ghUf_'a]h XYf'BUhi f'j YfVi bXYb'g]bX"; YgW\]W\hYb'ØVYf'Hfc``Yž'9`ZYb'i bX UbXYfY'a U[]gW\Y'K YgYb'g]bX'h]YZ']b'XYf'bcfk Y[]gW\Yb'?i `hi f j Yfk i fnY`h"'8]YgY':][i fYb'k YfXYb'cZh']b'@]hYfUhi fž'?i bgh'i bX HfUX]h]cbYb']bbYf\U`V'XYf'bcfk Y[]gW\Yb'; Ya Y]bgW\UZhYb XUf[YghY``h"

$6Y^{Y}YVhY'_i$ `hi $fY^{Y}GmaVc^{Y}$.

- Hfc``Y. 'Hfc``Y'g]bX'nYbhfU`Y':][i fYb'XYf'bcfk Y[]gW\Yb
 : c`_`cfY'i bX'k YfXYb'cZh'U`g'6YgW\ØhnYf'XYf'bUhØf`]W\Yb
 @UbXgW\UZhYb'XUf[YghY``h"
- 8 Yf'8 fUWX Yb Vcch. '8]YgY'hfUX]h]cbY``Yb'6cchY'gd]Y`hYb Y]bY'k]WXh][Y'Fc``Y']b'XYf'bcfX]gWXYb'GYYZU\fhi bX'g]bX \Yi hY'Y]b'Gma Vc`'ZØf'X]Y'bcfk Y[]gWXYb'K i fnY`b"
- ?i bgh'i bX' < UbXk Yf_. 'Bcfk Y[]gWXYg' < UbXk Yf_ž'k]Y XUg'VYfØ\a hY' < Uf`Y_]b!A i ghYfž'gd]Y[Y`h'X]Y'@]YVY'ni a

9]bZiggjcb?ibghibXAig]_

?i bgh'i bX'A i g]_'gd]Y`Yb'Y]bY'k YgYbh`]W\Y'Fc``Y']b'XYf
bcfk Y[]gW\Yb'?i `hi f"'Bcfk Y[Yb'\Uh'j]Y`Y'hU`Ybh]YfhY'?Øbgh`Yf
\Yfj cf[YVfUW\hž'i bX'X]Y'A i g]_gnYbY'fY]W\h'j cb'hfUX]h]cbY``Yb
Jc`_ga i g]_f]W\hi b[Yb'V]g'\]b'ni 'a cXYfbYa 'Dcd'i bX'FcW_"'8Ug
VYfØ\a hY'9Xj UfX'A i bW\!A i gYi a ']b'Cg`c']gh'Y]b'A i gg'ZØf
?i bgh`]YV\UVYf'i bX'V]YhYh'Y]bYb'9]bV`]W_']b'X]Y'K Yf_Y'XYg
VY_UbbhYb'A U`Yfg"

AcXYfbYAig]_ZYgh]jU`g.

- f(x) = f(x) + f(x) = f(x) + f(x) = f(x) + f(x) = f(x) + f(x) = f(x)
- 6Yf[YbZYgh. '8]YgYg': Ygh]j U``]b'6Yf[Yb'VY[Y]ghYfh'a]h Y]bYf'A]gW\ib['Uig'jYfgW\]YXYbYb'Aig]_[YbfYg'ibX n]Y\h'HUigYbXY'jcb'6YgiW\Yfb'Ub"
- G`chhgZ'Y``: Ygh]j U`. '8 Ug': Ygh]j U`']b 'HÔbgVYf[']gh VY_Ubbh'ZØf'gY]bY'Y]bn][Ufh][Y'?i `]ggY'i bX'j]Y`ZÀ`h][Y A i g]_dfc[fUa a Y"

HfUX]h]cbY``Y'bcfkY[]gWXY'?ØWXY

8]Y'bcfk Y[]gWXY'?ØWXY']ghYVYbZU``g'Y]b'k]WXh][Yf'HY]`'XYf _i `hi fY``Yb'\\XYbh]h\Ah"'6Ug]YfYbX'Ui Z'Zf]gWXYb'Ni hUhYb'Ui g'XYb i a `]Y[YbXYb'; Yk \AggYfb'i bX'`UbXk]fhgWXUZh`]WXYb'; YV]YhYb i a ZUggh'g]Y'Y]bY'J]Y`nU\`'j cb'fY[]cbU`Yb'; Yf]WXhYb\Z'X]Y'X]Y ; YgWX]WXhY'i bX'HfUX]h]cbYb'XYg'@UbXYg'k]XYfqd]Y[Y`b"'

 $Ni \ XYb \ VY_UbbhYghYb'; \ Yf]WXhYb'nÀ\ Yb.$

- ?`]ddZ]g_.`HfcW_Yb`[YgU`nYbYf`:]gW\ž`\UidhgÀW\`]W\ ?UVY`'Uiž'XYf`cZh`]b`hfUX]h]cbY``Yb`;Yf]W\hYb`jYfkYbXYh k]fX"
- @Udg_Ui g. '9]b '\Yfn\UZhYg'9]bhcdZ[Yf]W\h'Ui g'? UfhcZZY`bž
 : `Y]gW\i bX'; Ya ØgYžXUg'ZØf'j]Y`Y'Bcfk Y[Yf'bcghU`[]gW\Y

9f]bbYfi b[Yb'k YW_h"

• F Ôa a Y[fÔh. '9]b'X]W_Yf'6fY]'Ui g'GUi YffU\a]`W\ž'XYf'cZh ni 'VYgcbXYfYb'5b`ÀggYb'gYfj]Yfh'k]fX"

8]YgY'_i `hi fY``Yb'9`Ya YbhY'a UW\Yb'Bcfk Y[Yb'ni 'Y]bYa
Y]bn][Ufh][Yb'FY]gY`UbXž']b'XYa 'HfUX]h]cbYb'`YVYbX]['[Y\U`hYb
k YfXYb'i bX'[`Y]W\nY]h]['Y]b'FUi a 'ZØf'a cXYfbY'9]bZØggY
[YgW\UZZYb'k]fX"'8]Y'A]gW\i b['Ui g'; YgW\]W\hYž'?i bghž'A i g]_'i bX
i `]bUf]gW\Yb'HfUX]h]cbYb'V]YhYh'6Ygi W\Yfb'Y]bYb'h]YZYb'9]bV`]W
]b'X]Y'GYY`Y'XYg'@UbXYg'i bX'VYfY]W\Yfh'YXY'FY]gY'Xi fW\'XUg
k i bXYfgW\`ObY'Bcfk Y[Yb"

$DfU_h]gWXY'H]ddg'ZØf'Y]bY'fY]Vib[g`cgYFY]gY$

FY]gYnY]h

$HfUbgdcfha \dot{O}[\dot{M}_Y]hYb$

Bcfk Y[Yb']gh'[i h'j YfbYhnhž'i bX'Yg'[]Vh'a Y\fYfY'A Ò[`]WX_Y]hYbž

g]WX]a @UbX Zcfhni VYk Y[Yb.

- : `i [nYi [. ' \pm b`UbXgZØ[Y'j YfV]bXYb'X]Y'[fÒ» YfYb'GhÀXhY gW\bY``"
- 6U\b. '8]Y 'bcfk Y[]gW\Y '6U\b'V]YhYh'Y]bY 'a U`Yf]gW\Y FY]gY 'Xi fW\'X]Y '@UbXgW\UZhž'VYgcbXYfg'X]Y 'GhfYW_Y 'j cb Cg`c'bUW\'6Yf[Yb"
- 6i g. '6i ggY 'g]bX 'Y]bY '[Øbgh][Y 'A Ò[`]WX _Y]hž'j]Y `Y 'GhÀXhY i bX 'GY \ Ybgk ØfX][_Y]hYb 'ni 'YffY]WX Yb "
- 5i hc. '9]b 'A]Yhk U[Yb'[]Vh' ⇒ bYb 'X]Y': fY]\Y]hž'Ybh`Y[YbY
 'cfXY'i bX'UhYa VYfUi VYbXY'@UbXqW\UZhYb'ni 'Yf_i bXYb"

I bhYf i bZh

8]Y'5i gk U\`'Ub'I bhYf_ØbZhYb']b'Bcfk Y[Yb']gh'j]Y`ZÀ`h]["'CV'G]Y']bYa'
chY`ž'Y]bYa'; ÀghY\Ui g'cXYf'Y]bYa'hfUX]h]cbY``YbFcfVi Yf'fl]gWXYf\Ui gŁØVYfbUWXhYb'a ÒWXhYbž'X]Y'Cdh]cbYb'g]bXbU\Yni 'YbX`cg"'6Y`]YVhY'6i WXi b[gd`UhhZcfa Yb'k]Y'6cc_]b["Wca'cXYf'5]fVbV'V]YhYb'Y]bY'VfY]hY'DU`YhhY'Ub5b[YVchYbž'X]Y' \Rightarrow fYa'6i X[Yh'YbhgdfYWXYb"

?fYX]h_UfhYb i bX K A\fi b[

8]Y'cZZ]n]Y``Y'K À\fi b['Bcfk Y[Ybg']gh'X]Y'? fcbYb'fBC? Ł"' ₺ 'XYb a Y]ghYb'ghÀXh]gWXYb'; YV]YhYb'k YfXYb'? fYX]h_UfhYb'U_nYdh]Yfhž UVYf'Yg']gh'fUhgUa ž'Ui W 'Yhk Ug'6Uf[Y`X'ZØf'_`Y]bYfY'9]b_Ài ZY cXYf'`ÀbX`]WXY'; YV]YhY'a]hni bY\a Yb"'; Y`XUi hca UhYb'g]bX'k Y]h i YfVfY]hYhi bX'Yfa Ò[`]WXYb'Y]bZUWXY'6Uf[Y`XUV\YVi b[Yb"

GdfUWXY

⇒ Bcfk Y[Yb'k]fX'Bcfk Y[]gW\'[YgdfcW\Yb/'YXcW\'gdfYW\Yb'X]Y
a Y]ghYb'Bcfk Y[Yf'Ui W\'Z]Y» YbX'9b[`]gW\"'8]Yg'Yf`Y]W\hYfh'X]Y
?ca a i b]_Uh]cb'Yf\YV`]W\"'9]b'dUUf'[fi bX`Y[YbXY'bcfk Y[]gW\Y
5 i gXfØW_Y'_ÒbbYb'YXcW\'b]W\h'gW\UXYb.

- \bullet < U``C.'< Y]
- 8 Ub _Y. 'HU__

• 6]hhY. 'J Â f'gÁ'gb]``

; Ygi bX Y]hi bXG]WXYf Y]h

Bcfk Y[Yb'[]'h'U'g'g]WXYfYg'FY]gY`UbX'a]h'[Yf]b[Yf'?f]a]bU`]hÀh"

9g']gh'YXcWX'fUhgUa ž'X]Y'[fi bX`Y[YbXYb

G]WXYf\Y]hgj cf_Y\fi b[Yb'ni 'VYUWXhYbž'k]Y'XUg'5i ZVYk U\fYb'j cb

K YfhgUWXYb'Ub'Y]bYa 'g]WXYfYb'Cfh"8 Ug'ÒZZYbh']WXY

; Ygi bX\Y]hggmghYa ']gh'[i h'Ui g[YVUi hž'i bX'U`g'9I !6Øf[Yf'gc``hYb

G]Y'g]WXYfghY``Ybž'XUgg'G]Y'Y]bY'9< =7!? UfhY'fl9i fcdÀ]gWXY

?fUb_Ybj Yfg]WXYfi b[g_UfhYŁ'\UVYbž'i a 'Ni [Ub['ni bchk YbX][Yb'a YX]n]b]gWXYb'8]YbghY]ghi b[Yb'ni 'Yf\U`hYb"

; YgYhnYibXJcfgWXf]ZhYb

Bcfk Y[Yb`\Uh`ghf]_hY; YgYhnYž`]bgVYgcbXYfY`]b`6Yni [`Ui Z 5`_c\c`ž'8fc[Yb`i bX`JYf_Y\fgfY[Y`b"'8Ug': U\fYb`i bhYf 5`_c\c`Y]bZi gg`]gh`ghfYb[ghYbg`j YfVchYbž`i bX`Yg`[]Vh`\c\Y 6i »[Y`XYf`ZØf'; YgW\k]bX][_Y]hgØVYfgW\fY]hi b[Yb"'5W\hYb`G]Y XUfUi Zž`g]W\`Ub`X]Y`; YgW\k]bX][_Y]hgVY[fYbni b[Yb`ni `\U`hYb i bX`X]Y`JYf_Y\fgnY]W\Yb`ni `VYZc`[Yb"

JcfVYfY]hi b['Ui Z'XUg'K YhhYf

A cV]`]hÀh'j cf'Cfh

± XYb GhÀXhYb Bcfk Y[Ybg']gh XUg ÒZZYbh`]WXY J Yf_Y\fggmghYa
[i h Ybhk]W_Y`h"8]Y a Y]ghYb GhÀXhY V]YhYb 6i ggYž GhfU» Yb VU\b Yb
i b X : À\fYb ž i a X]Y 9f_i b Xi b['ni Yf`Y]WXhYfb": Øf ``Ub[Y
GhfYW_Yb '_ÒbbYb 'G]Y X]Y 'HfUa!'i b X I!6U\b!GmghYa Y 'bi hnYb"
± ``Àb X`]WXYfYb ; YV]YhYb '_ÒbbYb 'G]Y 'Ui WX j cb : À\fYb i b X

? `Y]bVi ggYb 'dfcZ]h]YfYbž'X]Y 'cZh'gW\k Yf'ni 'YffY]W\YbXY hci f]gh]gW\Y'CfhY'VYX]YbYb"

BchZU``bi a a Yfb

- =a 'BchZU``'_ÒbbYb'G]Y'X]Y'Zc`[YbXYb'BiaaYfb'jYfkYbXYb.
 - Dc`]nY]. '%%&
 - F Y hhi b [gX]Y b gh. '%%'
 - : Yi Yfk Y\f. '%%\$

 $A \]h'X]YgYb'dfU_h]gWXYb'H]ddg'g]bX'G]Y'VYghYbg'j cfVYfY]hYh'ZØf' \Rightarrow fY FY]gY'Xi fWX'XUg'k i bXYfgWX ObY'Bcfk Y[Yb''; Yb]Y»Yb'G]Y'X]Y VYY]bXfi W_YbXY'BUhi fž'X]Y'ZUgn]b]YfYbXY'?i `hi f'i bX'X]Y'\Yfn`]WXY ; UghZfYi bXgWXUZhž'X]Y'X]YgYg'@UbX'ni 'V]YhYb'\Uh'$

8 YhU]`g

6Ygi WXYb G]Y i bg Ui Z Y]bY!fY]gY "XY