

² DBJ. '> XYf '9i fc '= bj Ygh]h]cb 'Vf]b[h'XfY] 9i fc 'ZØf'X]Y 'K]fhgW UZh°

9]bY'Ghi X]Y'nY][hž'k]Y'XYf'ÒZZYbh`]WXY'BU\jYf_Y\f'XYb Hcif]ga i g'i bX'9]bnY`\UbXY`']b'8Yi hgWX`UbX'ZÒfXYfh'i bX j c`_gk]fhgWXUZh`]WXY'J cfhY]`Y'gWXUZZh"



8]Y'Ghi X]Y'nY][h'Ui Zž'XUgg'X]Y'X]fY_hY'i bX']bX]fY_hY K YfhgWX ÒdZ b['Xi fWX'X]Y'² DBJ!6fUbWXY'i bX'j Yfk UbXhY K]fhgWW UZhgVYfY]WW Y 'À\f`]WW 'Yhk U '&%'A]``]UfXYb '9i fc 'VYhfÀ[h" 6YgcbXYfg 'VYY]bXfi W_YbX ']gh'XYf '9]bZi gg 'XYg '2 DBJ 'Ui Z'XYb 9]bnY`\UbXY`ž'XYf 'À\f`]WW 'Yhk U 'ZØbZ'A]``]UfXYb '9i fc 'Ub K YfhgWW ÒdZi b['Xi fWW '2 DBJ!: U\f[AghY 'Yfn]Y`h" '8 UfØVYf '\]bUi g dfcZ]h]Yfh'XYf 'Hci f]ga i ggY_hcf'j cb 'Y]bYf'[i hYb'2 DBJ! 9ffY]WW VUf_Y]hž'k Ug'Ui Z'[YgWW AhnhY 'nk Ò`Z'A]``]UfXYb'9i fc 'A\f`]WW \]bUi g`Ài Zh"

$K \]WX h][Y j c \ gd Y hY$

9]b'k YgYbh`]WXYf'5gdY_h'XYf'Ghi X]Y']gh'XYf'Ni [Ub['ni 5fVY]hgd`AhnYbž'XYb'XYf'^2 DBJ'XYb'6YgWXÀZh][hYb'Yfa Ò[`]WXh'' 8]YgY'9ffY]WXVUf_Y]h'ZØ\fh'ni 'Y]bYf'K YfhgWXÒdZ b['j cb'fi bX'&& A]``]UfXYb'9i fc''5i WX'XYf'a a cV]`]YbgY_hcf'k]fX'hUb[]Yfhž'XU a cV]`]Yb']b'XYf'BÀ\Y'j cb'^2 DBJ!?bchYbdi b_hYb'\Ò\YfY HfUbgU_h]cbgdfY]gY'Yfn]Y'Yb'''8Yf'[YgWXÀhnhY'9]bZi gg'XYg BU\j Yf_Y\fg'Ui Z'X]YgYb'GY_hcf'VYhfÀ[h'fi bX'gYWXg'A]``]UfXYb 9i fc'''8]Y'9f[YVb]ggY'VY`Y[Ybž'XUgg'Y]bY'JYf`U[Yfi b['XYf\Yi h][Yb'JYf_Y\fg'Y]ghi b['XYg'^2 DBJ'Ui Z'a chcf]g]YfhYb \dank X]j]Xi U`j Yf_Y\f'A\f`]WX'ni gÀhn`]WX'Yhk U'bYi b'A]``]UfXYb'9i fc Ub'j c`_gk]fhgWXUZh`]WXYb'?cghYb'j Yfi fgUWXYb'k ØfXY''

86 FY[]c!7\YZ'\Jb'GW\]``]b['VY_f\AZh][hY'X]Y'Bchk YbX][_Y]h\b'XUg

2 DBJ!5b[YVch\b']bgVYgcbXYfY']a '`\AbX`]W\Yb'FUi a \b'Ui gni VUi Yb''

9f'VYhcbhY\b'XUgg'X]Y'\c\Yb'j c`_gk]fhgW\UZh`]W\Yb'J cfhY]`Y'Y]bYf

I bhYfgh\Omegahni b['i bX'\bj Ygh]h]cb']b'XYb'^2 DBJ'b]W\h'][bcf]Yfh

k YfXYb'X\OmegafZYb'''8]Y'Ghi X]Y'nY][h'Ui Z\b'XUgg'XYf'^2 DBJ'b]W\h'bi f

DYbX`Yf'Ybh`UghYh\b'gcbXYfb'Ui W\'ni f'Gh\Af_i b['j cb'9]bnY`\UbXY`
i bX'Hci f]ga i qgY_hcf'VY]hf\A[h''

:]bUbn]Yfi b[g\YfUi gZcfXYfi b[Yb

 $\label{thm:converse} Hfchn: XYf: dcg]h]j: Yb: 9f[YVb]ggY:ghY \ h: XYf: OZZYbh:]WXY:BU\ j: Yf_Y\ f]b: 8Yi: hgWX: UbX:j: cf:Yf\YV:]WXYb: Z]bUbn]Y: Yb: < YfUi: gZcfXYfi: b[Yb": 9f:[]:h:U:g:WX:fcb]gWX:i: bhYfZ]bUbn]Yfhz:k: cVY]: Yhk: U:X]Y: < A: ZhY: XYg: 6Yhf]YVg: Xi: fWX:: U\f[UghY]bbU\a:Yb:i: bX:FY[]cbU:]g]Yfi: b[ga:]hhY: XYg: 6i: bXYg:[YXYW_h:k:]fX":A:Y\f:U:g:Y]b: 8f]hhY: XYf: 8Yi: hgWX:Yb:]gh: a:]h: XYf: 5bV]bXi: b[:XYg:^2:DBJ:i:bni:Zf]YXYbz: VYgcbXYfg:]b$

`ÀbX`]WXYb'; YV]YhYb"'8]YgY'I bni Zf]YXYb\Y]h'_ÒbbhY'`Ub[Zf]gh][bY[Uh]j Y'5i gk]f_i b[Yb'Ui Z'X]Y'K YfhgWX ÒdZi b['i bX'X]Y'dcg]h]j Y 9bhk]W_`i b['XYf'j c`_gk]fhgWXUZh`]WXYb'9ZZY_hY'\UVYb"'9]b][Y 6i bXYg`ÀbXYf'n]Y\Yb'VYfY]hg'Y]bY'FYXi n]Yfi b['XYg'² DBJ! 5b[YVchg'Ui Z[fi bX'ZY\`YbXYf'A]hhY`']b'6YhfUWXh"

9gʻghY\hʻZYghžʻXUggʻXYfʻ²DBJʻY]bYʻGW\`ØggY`fc``Yʻ]aʻXYi hgW\Yb K]fhgW\UZhggmghYaʻgd]Y`h"ʻ9gʻV`Y]VhʻYXcW\`UVni kUfhYbžʻcVʻX]Y dc`]h]gW\Ybʻ9bhgW\Y]Xi b[ghf\[Yf`X]Y`YfZcfXYf`]W\YbʻGW\f]hhY i bhYfbY\a Ybži aʻXYbʻ²DBJʻ`Ub[Zf]gh][ʻbUW\\U`h][ʻni 'Z]bUbn]YfYb i bX`Ui gni VUi Ybži aʻX]Yʻj c`_gk]fhgW\UZh`]W\YbʻJcfhY]`YʻUi W\']b Ni_i bZh'ni 'g]W\Yfb"

8]Y'j c``ghÀbX][Y'5bU`mgY'XYf'Ghi X]Y'_Ubb'Ui Z'HU[Yggd]Y[Y``i bX N8: 'Y]b[YgY\Yb'k YfXYb"

8 YhU]`g	
Cfh	8 Yi hgWX`UbX
EiY``Yb	• k k k "hU[Yggd]Y[Y`"XY
	• k k k "nXZ"XY

6Ygi WXYb G]Y i bg Ui Z Y]bY!fY]gY "XY