

I a k Y`hZfYi bX`]W\Yf`Hci f]ga i gʻUi ZʻXYb ; U`½dU[cg

: Øf'j]Y`Y']gh'Yg'Y]b 'HfUi a n]Y`. 'X]Y'i bVYfØ\fhYb ⇒gY`b'XYf
; U'½dU[cg! ⇒gY`bžNi Zi W\hgcfh'ZØf'Y]b][Y'XYf
Y]bn][Ufh][ghYb'i bX'gY`hYbghYb'H]YfUfhYb'XYf'K Y`h' gc
XUW\hYb'k]f'ni a]bXYgh"'9g'[UV'g]W\Yf`]W'i bVYfØ\fhY'HY]`Y
XYf' ⇒gY`bž'UVYf'Yg'[UV'Ui W\'GhfU» YbUfVY]hYb']b'Di Yfhc
5mcfUž'nYfVfcW\YbY'6]YfZUgW\Yb'Ua '7Yffc'H]'YfYhUgž
D`Ugh]_Z'UgW\Yb']b'XYf'6i W\h'j cb'Hcfhi [U'i bXž'Ua
hfUi f][ghYbž'Y]b'FcVVYbVUVmž'XUg'a]h'Y]bYa 'D`Ugh]_`OZZY`
gd]Y`hY"'K]f'_Ua Yb'b]W\h'i a\]b'ni 'ZfU[Ybž'cV'X]Y
; U'½dU[cg'VYggYf']b'Fi\Y'[Y`UggYb'k YfXYb'gc'`hYbž
i bVYfØ\fhi bX'i bVYZYW_h'j ca 'Hci f]ga i g"'K]f'g]bX'b]W\h']b
XYf'@U[Yž'ni _ØbZh][Yb'Hci f]ghYb'j cb'Y]bYa '6Ygi W\
UVni fUhYbž'UVYf'k]f'_ObbYb' ⇒bYb'nY][Ybž'k]Y'/ Ua d∧Y``]d/

: Øf'j]Y`Y']gh'Yg'Y]b'HfUi a n]Y`.'X]Y'i bVYfØ\fhYb'±gY`b'XYf
; U`½dU[cg!±gY`bž'Ni Zi Wkhgcfh'ZØf'Y]b][Y'XYf'Y]bn][Ufh][ghYb'i bX
gY`hYbghYb'H]YfUfhYb'XYf'K Y`h' 'gc'XUWkhYb'k]f'ni a]bXYgh"'9g'[UV
g]WkYf`]Wk'i bVYfØ\fhY'HY]`Y'XYf'±bgY`bž'UVYf'Yg'[UV'Ui Wk
GhfU» YbUfVY]hYb']b'Di Yfhc'5mcfUž'nYfVfcWkYbY'6]YfZUgWkYb'Ua
7Yffc'H]YfYhUgž'D`Ugh]_ZUgWkYb']b'XYf'6i Wkhj cb'Hcfhi [U'i bXž
Ua 'hfUi f][ghYbž'Y]b'FcVVYbVUVmž'XUg'a]h'Y]bYa 'D`Ugh]_`ÒZZY`
qd]Y`hY"

K]f'_Ua Yb'b]W\hi a \]b'ni 'ZfU[YbžcV'X]Y'; U'\\dU[cg'V\ygg\f']b

Fi\\Y'[Y`Ugg\b'k\YfX\Yb'gc``h\Ybži\b\V\fØ\fh'i\b\X'i\b\V\Z\Y_h'j\ca

Hci f]ga i g"'K]f'g]b\X'b]W\h']b'\X\Yf'@U[\Yž'ni_Øb\Zh][\Yb'\Hci f]gh\Yb'j\cb

Y]b\Ya '6\Ygi W\`U\Vni\f\Uh\Ybž'\U\Y\f'\k\]f'_Obb\Yb'\⇒\b\Yb'\n\Y][\Ybž'\k\]\Y'G]\Y

X]\Yg'a O[`]\W\gh'i\a\k\Y`\hg\W\cb\Yb\X'\hi\b'_Obb\Yb''

 $A\ UWXYb\ G]Y\ g]WX\ ini\ bAWX\ gh\ VYk\ i\ ggh\ XUgg\ Hc\ i\ f]gh\ Yb\ bi\ f\ ini\ Y]bYa$

 $k \]bn][\ Yb'HY]``XYf' + bgY`b'ib_cbhfc``]YfhYb'Ni [\ Ub[\ ``\UVYb''-+DfcnYbhXYf' + bgY`g]bX'Xi fW. `XYb'BUh]cbU`dUf_'[\ YgW. \emptysethnh'ibX'X]Y + $'fY[\]ghf]YfhYb'6Ygi W. YfcfhY'ia ZUggYb'bi f'z%'DfcnYbh'XYf @UbXa UggY'0%Q'Nk Y]hYbg'a igg'YXYf'Hcif]gh'%$$'8c``Uf'9]bhf]hh nU\`Yb''K À\fYbX'Y]b'[fc\ Yf'HY]``XYf'Hcif]ga ig[Y`XYf'UiZ'XUg YW'UXcf]Ub]gW.Y': Ygh`UbX'Z]Y\ hz'hfU[Yb'YXYg'\J\f'Yhk U'*' A]``]cbYb'I G!8c``Uf'nif``c_U`Yb'K]fhgW.UZhibX'XYb BUhifgW.ihnVYa \emptyset\ib[Yb'UiZ'XYb'\DgY`b'VY]'`&Q'K Ybb'G]Y'g]W. gW.`]Y\"]W.'\DfYg': i\ UVXfi\ W_g'YXYfnY]h'VYk iggh'g]bXz'_\ObbYb'G]Y X]Y'5i gk]f_i\ b[Yb'\DfYg'6Ygi\ W.g'Yf\YV`]W.'\fYXin]YfYb'''<]Yf']gh k]Y''$

9fk À[Yb'G]Y'Y]bY'9f_i bXi b['Ui Z'XYa @UbXk Y[

8 Yf'B Uh]cbU`dUf_'VYgW\fÀb_h'X]Y'; fÒ» Y'j cb'6cchYb'Ui Z'%\$\$
DUggU[]YfYž'UVYf'gY`Vgh'%\$\$'DYfgcbYb'_ÒbbYb'Y]bYb'GhfUbX'VY]a
5i gghY][Yb'ØVYf`UghYb"'5bghUhh'Y]bY'[fc» Y'?fYi nZU\fh'ni
i bhYfbY\a Ybž'gc``hYb'G]Y'X]Y'\DgY`b'Ui Z'XYa '@UbXk Y['Yf_i bXYb
i bX'ÒZZYbh`]W\Y': À\fYb'ni 'XYb'UbXYfYb'\DgY`b'bY\a Yb"'8]Yg'\Uh
XYb'ni gÀhn`]W\Yb'J cfhY]`ž'XUgg'Yg'ghUhh'Y]bYg']bhYfbUh]cbU`Yb
?fYi nZU\fhi bhYfbY\a Ybg'ni f'`c_U`Yb'K]fhgW\UZh'VY]hfÀ[h"

$6Y]VYbG]YVY]XYbVYkc\bhYb=bgYb$

9]b 'HY] `XYf'JYf`cW_i b['XYf'; U`\\dU[cg!\DgY`b 'VYghY\h'XUf]b\Z'XUgg [fc\\Y'HY]`Y'j cb 'Hci f]ghYb'i bVYf\[Ø\fh'g]b\X' 'UVYf'k Ybb'G]Y'\X]Y i bVYk c\bhYb'\DgY`b'VYgi W\Yb\Z'g]b\X'g]Y'b]W\h'a Y\f'j cb'Hci f]ghYb i bVYf\[Ø\fhf'\Z'''VYk c\bhYb'\DgY`b'GUbhU'7fi n\Z'GUb'7f]ghcVU`\Z'\DVY`U'i b\X': `cfYUbU'i b\X']b'\XYb'i a `]Y[Yb\XYb'; Yk\AggYfb\Z'U`gc Yfk\A[Yb'G]Y\Z'bi f'\X]YgY'ni 'VYgi W\Yb''

K UggYf gdUfYb i a 'YXYb DfY]g

K UggYf']gh'Ui Z'XYb' +bgY`b'gY\f'_cghVUfž'U`gc'ghY``Yb'G]Y'g]W\Yfž
XUgg'G]Y'X]Y'; fi bX`U[Yb'UVXYW_Yb.'_i fnY'8i gW\Yb'i bX
_Y]bYgZU``g'6ÀXYfž'XfY\Yb'G]Y'XYb'K UggYf\U\b'VY]a

NÀ\bYdi hnYb'ni 'i bX'j Yfk YbXYb'G]Y' \Rightarrow fY'Hc]`YhhYbgdØ`i b[gdUfgUa "

5 V ZU``ZUWX [YfYWXh'Ybhgcf[Yb

6YZc`[Yb'G]Y'X]Y'Ei UfUbhÀbYj YfZU\fYb

B]WkhfUi WkYb

FUi WXYb ii bX '@U[YfZYiYf'g]bX ']a 'BUh]cbU`dUf_'jYfVchYb"'G]Y 'g]bX Y]bY'; YZU\f'ZØf': `cfU'i bX': Ui bUž'U`gc'i bhYf`UggYb'G]Y'Yg"

$K \dot{A} \ \dot{g} G Y 5_h j] h \dot{h} h Y b g c f [Z \dot{A} h] [U g$

%[`]WXY'5fh'XYg'5b[Y`bg'jcb'5igZi[gVcchYb'Uig']gh'jYfVchYb"
5i »YfXYa 'g]bX'a chcf]g]YfhYf'K UggYfgdcfhž'A]b]!I!6cchY'ibX
: `i[hcif]gaig']a 'BUh]cbU`dUf_'b]WXh'Yf`UiVh''K Ybb'G]Y'X]Yg
gY\Ybž'a Y`XYb'G]Y'Yg'XYa 'BUh]cbU`'DUf_'GYfj]WY''fHWX'_ca a Y
Uig'9Ugh'@cbXcbž'U`gc'V]b']WX'_Y]b'; fUgž'UVYf'a UbWXa U`']gh'Yg
XUg'F]WXh][Y"Ł

? Ui ZYb G]Y Gci j Yb]fg gcf[ZÀ`h][

GcijYb]fg'Uig'gWkkUfnYf'?cfU``Yž'AigWkY`bž'Ji`_Ub[YghY]bž
H]YfhY]`Yb'ibX'Y]b\Y]a]gWkYb'<Ò`nYfb'XØfZYb'b]Wkh'[Y_UiZh
ibX#cXYf'jcb'XYb'+bgY`b'a]h[YbcaaYb'kYfXYb"'KYbb'G]Y'X]Yg
gY\Ybž'aY`XYb'G]Y'Yg"

6 Y] V Y b G] Y U i Z a U f _] Y f h Y b K Y [Y b

Ni a 'GW\i hn'XYf'H]Yfk Y`h'g]bX'K UbXYfk Y[Y'a Uf_]Yfh"'K Ybb'G]YUVV]Y[Ybž'k YfXYb'G]Y'k U\fgW\Y]b`]W\'XUg'bUhØf`]W\Y; `Y]W\[Yk]W\h'Ui Z'XYb'\DgY`b'ghÒfYbž'U`gc'k]XYfghY\Yb'G]Y'XYf
J Yfgi W\i b["'; YV]YhY']bbYf\U`V'XYg'BUh]cbU`dUf_g'_ObbYb'bi f
a]h'Y]bYa '`]nYbn]YfhYb'; U`\dU[cg!BUh]cbU`dUf_ZØ\fYf'VYgi W\h
k YfXYbž'j Yfgi W\Yb'G]Y'U`gc'b]W\hž'X]YgY'Ui Z'Y][YbY': Ui gh'ni
VYhfYhYb"

< U`hYb'G]Y'5VghUbX'ni'K]`Xh]YfYb

 $6Yf\emptyset \ fYb \ G]Y \ g]Y \ b]WXh \ bX \ \ UggYb \ G]Y \ g]WX \ b]WXh \ VYf\emptyset \ fYb \ "$

$: \emptyset hhYfb G]Y X]Y K]`Xh]YfY b]WXh$

K]f'k i fXYb'VY]a ': fØ\ghØW_'j cb'Y]bYf'; fi ddY':]b_Yb'VY[`Y]hYh i bX'k UfYb'j Yfgi W\hž'g]Y'a]h'i bgYfYb'?fØa Y`b'ni 'ZØhhYfbž k]XYfgYhnhYb'i bg'UVYf"'8]Y'H]YfY'_ObbYb'g]W\gY`Vgh'YfbÀ\fYb" JYfÀbXYfY'b]W\h'XUg'bUhØf`]W\Y'; `Y]W\[Yk]W\h'Ui Z'XYb'=bgY`b"

"U"h1 H]Yfk Y"h!XYf!; U"UdU[cg!<math>=bgY"b!: chc[fUZ]Y 25h"Ug"/ '6cchg

JYfk YbXYb'G]Y'_Y]bY'6`]hnZchc[fUZ]Y'Ui Z'X]Y H]Yfk Y`h

8]Yg'_Ubb']\fY'bUhØf`]W\Y'@YVYbgk Y]gY'j YfÀbXYfb"'DfcZYgg]cbY``Y : chc[fUZYb'i bX':]`a Ya UW\Yf'VYbÒh][Yb'Y]bY BUh]cbU`dUf_[YbY\a][i b["

JYfk YbXYb G]Y bi f Ui hcf]g]YfhY

7Ua d]b[d`ÀhnY

G]Y'a ØggYb'(, 'Ghi bXYb'j cf'XYa '5i ZVUi 'XYg'@U[Yfg'Y]bY; YbY\a][i b['Y]b\c`Yb"'K YbXYb'G]Y'g]WX'j cf'XYa '6Ygi WX'Ub XYb'BUh]cbU`'DUf_'GYfj]WY"

6 Ygi WX Yb G]Y X]Y = bhYfdfYhUh]cbgnYbhfYb

JYfVf]b[Yb'G]Y'NY]h'VY]'[Ya Y]bbØhn][Yb'Cf[Ub]gUh]cbYb'k]Y'XYa 7\Uf`Yg'8Ufk]b'FYgYUfW\'7YbhYfž'ia'a Y\f'ØVYf'X]Y'; YgW\]W\hY XYf'\deltagY`b'ni 'YfZU\fYb"'8Ug'\deltahYfdfYhUh]cbgnYbhfia'Ui Z'GUb 7f]ghcVU`']gh'VYgcbXYfg'[ihž'U`gc'bY\aYb'G]Y'g]W\'Yhk Ug'NY]hž'ia Xcfh\]b'ni '[Y`Ub[Yb"

K Ybb'G]Y'Y]b'< chY`fcXYf'Y]bYb'UbXYfYb'5bV]YhYfŁ'Vi WXYbž k YfZYb'G]Y'Y]bYb'6`]W_'Ui Z'XYggYb'I a k Y`hVYa Ø\i b[Yb"'FcmU` DU`a 'Ui Z'GUbhU'7fi n'i bX'7UgU'C di bh]U'Ui Z'GUb'7f]ghcVU``g]bX g]WXYfY'K YhhYbž'k Ybb'G]Y'XUg'6i X[Yh'\UVYb"'K Ybb'G]Y'Yhk Ug gY\Ybž'XUg'_b]fgWXhž'V`Y]VYb'G]Y'b]WXhighi a a "'5i Z'GUb'7f]ghcVU` gU\Yb'k]f'Y]bYb'6i g'a]h' 9Wc!: f]YbX`m'Hci fg 'Ui Z'XYf'GY]hYž'XYf %\$'A]bi hYb'`Ub['a]h'`Ui ZYbXYa 'A chcf'gh]``ghUbX"'K Ybb'G]Y']b Y]bYa 'gc`WXYb'6i g'g]hnYbž'gU[Yb'G]Y'Yhk Ug°

< Y V 'Y g 'Ui Z

 $K Ybb `G]Y `Y]bY `D`Ugh]_Z UgWk Y `Ua `GhfUb X `c XYf`A \varnothing```Ua GhfUb X `gY \ Yb ž'k YfZYb `G]Y `g]Y `]b `Y]bYb `A \varnothing``Y]a Yf" `G]Y k YfXYb `=\ fYb `HY]``XUni `VY]hfU[Yb ž'X]Y `GWk Ob \ Y]h`XYf `=\bgY`b `ni Yf\ U`hYb"$

 $GWX^{]}Y \gg ^{]}WX Z^{'}K Y b b ^{'}G]Y ^{'}b]WX h ^{'}K]f_^{]}WX ^{'}Ub ^{'}XY f B Uhi f ^{'}bhYfYgg]Yfh ^{'}g]b X Z^{'}n]Y \ Y b ^{'}G]Y ^{'}Yg ^{'}b 6 Y hf UWX hZ^{'}b]WX h ^{'}X c fh \]b ^{'}ni ^{'}[Y \ Y b ^{'}B]WX h ^{'}X c fh \]b ^{'}ni ^{'}[Y \ Y b ^{'}B]WX h ^{'}X c fh \]b ^{'}ni ^{'}[Y \ Y b ^{'}B]WX h ^{'}X c fh \]b ^{'}ni ^{'}[Y \ Y b ^{'}B]WX h ^{'}X c fh \]b ^{'}ni ^{'}[Y \ Y b ^{'}B]WX h ^{'}X c fh \]b ^{'}ni ^{'}[Y \ Y b ^{'}B]WX h ^{'}X c fh \]b ^{'}ni ^{'}[Y \ Y b ^{'}B]WX h ^{'}X c fh \]b ^{'}ni ^{'}[Y \ Y b ^{'}B]WX h ^{'}X c fh ^{'}Y b ^{'}Y b$

9g'VYghY\h'X]Y'; YZU\fž'XUgg'X]Y'; U'½dU[cg! \pgY`b'ni 'Y]bYa
A UggYbhci f]ga i gn]Y`'k YfXYbž'b]W\h'ZØf'BUhi f`]YV\UVYfž'gcbXYfb
bi f'ZØf'A YbgW\Ybž'X]Y'Y]bYb'gW\ÒbYb'I f`Ui V'j YfVf]b[Yb
a ÒW\hYb"'K Ybb'G]Y'XUg'g]bXž'gc``hYb'G]Y'ghUhhXYggYb'k cUbXYfg
\]b[Y\Yb"

 $9fZU\fYbG]YaY\f.[UUUU[cg"cf[$

@Y]hV]`X.~5h`Ug~/~6cchg

•

8 YhU]`g

6Ygi WXYb G]Y i bg Ui Z Y]bY!fY]gY "XY