

9WX hY'< cVV]h! < Ài gYfž']b'XYbYb'a Ub hUhgÀWX`]WX'ØVYfbUWX hYb'_Ubb

8]YgY'JYfÀbXYfi b[Yb'Yfa i h][hYb'i bgž']b'Y]bYa 'ZUgh VY]gd]Y``cgYb'5i ga U»'ØVYf'i bgYfY'@YVYbgk Y]gY'bUW\ni XYb_Ybž i bX'j]Y`Y'j cb'i bg'[Y`cVhYbž'bUW\\U`h][Yf'ni '`YVYbž'ni 'YggYb'i bX ni 'fY]gYbžgcVU`X'g]W\'X]Y'8]b[Y'k]YXYf'bcfa U`]g]Yfh'\UVYb"'BifkYb][Y'jcb'ibg'g]bX'VYfY]hž'@lig\chY`g'[Ubn'UiZni[YVYbž'UVYf]a a Yf'a Y\f'jcb'ibg'YbhgW\Y]XYb'g]W\'ZØf'BUhif\ÀigYf"

@Ui h'XYa 'b]YXYf`ÀbX]gW\Yb'GdYn]U`]ghYb'BUhi fY"\ci gY'g]bX'X]Yg CfhYž'Ub'XYbYb'G]Y'g]W\' j cb'⇒\fYa '[YgW\ÀZh][Yb'5``hU[ni fØW_n]Y\Yb'i bX'Y]bZUW\']ba]hhYb'XYf'BUhi f'YbhgdUbbYb _ÒbbYb "

 $\begin{tabular}{ll} J cb "]Xm``]gWXYb' < \varnothing hhYb ``Ua ``K UggYf`V]g``\]b `ni ``bYhni bUV\\. Ab[][Yb K UbXYf\\. \ØhhYb ``Y[Yb `X]YgY' < Ai gYf`K Yfh`Ui Z'B UWX\U`h][_Y]h'' B Uhi fY"\ci gY`ni a `6Y]gd]Y``\@VYfdf\@Zh`gY]bY`9]bhf\A[Y`i bX``Y\bh U``Yg`UV*zk Ug`ni ij c``*z`_ca a Yfn]Y```cXYf`_cbgi a cf]Ybh]Yfh`]gh'' \=\fY < c\text{\omega}bi b[`]gh`Yg*z`Y]bY`6Yk U\fi b[`XYf`B Uhi f`ni `Z\OfXYfb'' $HUhg\AWX\`]WX`gdYbXYb`g]Y`Z\@f``YXY`6i WXi b[*z`X]Y`\@VYf`]\fY`K YVg]hY [Yh\Ah][h`k]fX*z`\%' `Ub`B Uhi fdfc`\Y_hY''\6]g`\Yi hY`\UVYb`g]Y`a Y\f`U`g Y]bY`A]``]cb`6\Ai a Y`]b`A UXU[Ug_Uf*z`HUbgUb]U`i bX`6i f_]bU`: Ugc [Yd\text{\omega}Ubnh`i bX`UfVY]hYb`XYfnY]h`XUfUb*z`X]Y`6\AfYbghYfV`]WX_Y]h`i bX ?cb\text{\omega}]_hY`]b`XYf`FY[]cb`5Vfi nnYb`]b`\\=\U`]Yb`ni `fYXi n]YfYb'' $$$$

I bbÒh]['ni 'Yfk À\bYbž'XUgg'BUhi f\Ài gYf'dYfZY_h'ZØf'Ci hXccf! @]YV\UVYf'g]bX"'BUWXZc`[YbX'ghY``Yb'k]f'%\$'VYnUi VYfbXY'YWXhY <cVV]h!<Ài gYf'j cfž'X]Y'G]Y'\Yi hY'Vi WXYb'_ÒbbYb"

YWX hY'< cVV]h\ Ài gYf']b'Y]bYa'l B9G7C! FYgYfj Uh'

8]YgY'YW\hYb'<cVV]h!<\Ai gYf'VYZ]bXYb'g]W\']b'Y]bYa 'I B9G7C!
6]cgd\AfYbfYgYfj Uh']ba]hhYb'j cb'K]YgYbž'6\Ai a Yb'i bX
K Y]bfYVYbž'X]Y'hmd]gW\'ZØf'XUg'GUUf`UbX']a 'GØXk YghYb
8Yi hgW\`UbXg'g]bX'''8]Y'; \AfhYb'fYdf\AgYbh]YfYb'XUg'fY]W\Y'; fØb
XYf'; Y[YbXž'j cb'K]YgYb'ØVYf'K \A`XYf'V]g'\]b'ni 'CVgh[\AfhYb''

BUhi f"<Ui g'9WkhY'<cVV]h\Ài gYf']b'Y]bYa 'I B9G7C! 6]cgd\ÀfYbfYgYfj Uh

8]Y'Ui g'BUhi f\c`n'[YVUi hYb'<Ài gYf'V]YhYb'XYb'7\Ufa Y'Y]bYg `ÀbX`]WXYb'Ni Zi WXhqcfhqž'UVYf'XYb'?ca Zcfh'Y]bYg'a cXYfbYb $< chY^g. K @5Bž' < Y]nib["ibX"HJ"]$

± XYf'BÀ\Y'VYZ]bXYb'g]W\X]Y'GUUf`UbXh\Yfa Y'gck]Y'Y]b'BYhn'[ih Uig[YVUihYf'K UbXYf!'ibX'FUXkY[Y"'8Yf'Hcif]gaig'ghY\h']a
A]hhY`dib_h'XYf'BUW\\U`h][_Y]hgghfUhY[]Y'ibX'VYgW\fÀb_h'g]W\'UiZ
Y]bY', VYf[Ub[gncbYž'gcXUgg'[fc»Y'HY]`Y'XYg'FYgYfjUhg
ib[YghÒfh'V`Y]VYb''Y]b'dYfZY_hYf'Cfh'nia'5iZUhaYb"

6i WXYb G]Y X]YgYg B Uhi f\Ui g]b '? Y]bV`]hhYfgXcfZ

< Ài gWX Yb]a 'K U`X

 $8] YgY `hmd] gWX Y `K U`X \ \emptyset hhY `] gh`Y] b `VYnUi VYfbXYg `Gh\emptyset W \underline{} \\ gWX k YX] gWX Yb `8 Yg] [bg " 8 Ug `Y WX hY ` < c V V] h! < Ui g `h\ fc bh`Ui Z`Y] b Ya j cb `D] b] Yb `i a [YVYbYb ` < Ø[Y``] b `XYf`k] bn] [Yb `C fhgWX UZh': i fi XU` fl($+ '9] bk c \ bYfŁ] b `XYf`Dfcj] bn`8 U`UfbU`] b `A] hhY`gWX k YXYb"$

B Uhi f" < Ui g'9]b Y 'hmd]gW\ Y' < ØhhY']a 'K U`X

A]h": i » $VcXYb\Y$]ni b['i bX'Y]bYa '< c`ncZYb']gh'XUg'a U`Yf]gW\Y BUhi f\Ui g'XYf'< Ò\Ydi b_h'j cb'< m[[Y"'5i Z'Y]bYa 'Y\Ya U`][Yb

6Ui Yfb\cZ[Y`Y[Ybž'V]YhYh'Yg'X]Y'A Ò[`]W_Y]hž'9`W\Yž'FY\Yž : ØW\gY'i bX'j]Y``Y]W\h'gc[Uf'Y]bYb'@ W\g'ni 'VYcVUW\hYb"

8 U`UfbU'gY`Vgh'gc```GWk YXYb']a 'A]b]Uhi fZcfa Uh'gY]b"'6Yf[Yž GYYbž'hfUX]h]cbY``Y'fchY'< ØhhYbž'gUa]gWkY'?i `hi f'i bX < UbXk Yf_ghfUX]h]cb'g]bX'\]Yf'X]Wkh'[YXfÀb[h'i bX'V]YhYb'j]Y`Y 5_h]j]hÀhYbž'k Ybb'G]Y'g]Wk'ØVYf'X]Y'[Ya Øh`]WkYb'; fYbnYb'j cb: i fi XU``\]bUi gk U[Yb"

6i WXYb G]Y X]YgYg B Uhi f\Ui g]b : i fi XU`

< c`nfUi WX ii bX J c[Y`[Ynk]hgWXYf</pre>

<c`nfUi WX i bX'Jc[Y`[Ynk]hgWXYf'_ca a Yb'Y]bYa 'VY]a 'YfghYb
5bV`]W_'X]YgYg'[Ya Øh`]WXYb'<cVV]h\Ui gYg']b'XYf'FY[]cb
BcfXVfUVUbh']b'XYb'B]YXYf`UbXYb']b'XYb'G]bb''' \(\delta\) Y]b'gYWXg'A YhYf
`Ub[Yg': Ugg'[YgWXb]hnhž'VYZ]bXYh'Yg'g]WX'Ui Z'%ž)' < Y_hUf'@UbXž'j cb
XYbYb'Y]b'\U`VYf'< Y_hUf'XYf'BUhi f'j cfVY\U`hYb']gh'''9]b
5a d\]V]YbVYW_Yb'V]YhYh'Y]bYb'Ni Zi WXhgcfh'ZØf'Y]b\Y]a]gWXY
K]`Xh]YfYž'k À\fYbX'XYf'; UfhYb'`c_U`Y'DfcXi _hY'k]Y'CVgh'i bX
A Ufa Y`UXY'gYfj]Yfh''</pre>

 $B\ Uhi\ f''< Ui\ g`8]YgYg`[Ya\ \emptyseth`]W\&Y`< cVV]h\&Ui\ g`]gh`]b`Y]b`gYW\&g`A\ YhYf`Ub[Yg`:\ Ugg`[YgW\&b]hnh$

± XYf'k Y]hYfYb'l a [YVi b['j cb'BcfXVfUVUbh'VYZ]bXYb'g]W. Y]b B Uhi fgWli hn[YV]Yhi bX'X]Y'\]ghcf]gWlYb'GhÀXhY'6fYXU'i bX'8Yb 6cgWl"8]Y'Dfcj]bn']gh'XYf'; YVi fhgcfh'j cb'J]bW'bh'j Ub'; c[\žXU\Yf'k]fX'\]Yf'Ui Wl'j]Y`'?i bgh'i bX'?i `hi f'[YVchYb"

6i WXYb G]Y X]YgYg B Uhi f\Ui g]b ; Ya cbXY

ZUi `Y 5 V Y b X Y 'Ua 'K Ugg Y f

@ÒZZYfbž'GdYW\hYb'i bX'9i `Yb"'9g']gh'VY_Ubbhž'XUgg'Ui W\': ØW\gY Ui ZhUi W\Yb"

BUhi f"<Ui gʻ9fk UfhYbʻG]Yʻ[Ya Øh`]WkYʻ5VYbXYʻUa ʻK UggYfʻa]h 6Ygi WkYbʻj cbʻ6i ggUfXYbžʻ@ÒZZYfbʻi bXʻ9i `Yb

8 Yfi a `]Y[YbXY'K U`X'_Ubb'Ui Z'Y]bYf'FY]\Y'j cb'K UbXYf!'i bX FUXk Y[Yb'Yf_i bXYh'k YfXYbž'k À\fYbX'Y]b'bU\Y'[Y`Y[YbYg Ci hXccf!NYbhfi a '?`YhhYfb'i bX'?Ubi ZU\fYb'UbV]YhYh''9hk Ug k Y]hYf'YbhZYfbh'`]Y[h'9`mg]i a 'f&'_a Łž'Y]bYg'XYf'[fÒ» hYb'GdUg']b XYb'B]YXYf`UbXYbž'k À\fYbX'FchhYfXUa 'bi f'Y]bY'%)!a]bØh][Y 5i hcZU\fh'YbhZYfbh']gh"

6i WXYb G]Y X]YgYg B Uhi f\Ui g]b 6Yf[gWXYb\cY_

9]b HfUi a ZY X

8]YgYg'W\Ufa UbhY'<cVV]h\Ui g'VYZ]bXYh'g]W\'Ui Z'Y]bYa
6Ui Yfb\cZ']b'XYf'FY[]cb'I hfYW\h']b'XYb'B]YXYf`UbXYb'''?Ø\Y
[fUgYb'Ui Z'XYf'K]YgYž'J Ò[Y`'i bX': fÒgW\Y'nk]hgW\Yfb'ni Zf]YXYbž
k À\fYbX'Y]bY'GW\UZ\YfXY'X]YgY']Xm``]gW\Y'GnYbYf]Y
j Yfj c``ghÀbX][h"

BUhi f"<Ui g'8]Y'8Àa a Yfi b['Vf]W\h'ØVYf'X]YgYg'<cVV]h\Ui g'Ua 6Ui Yfb\cZ'\YfY]b

8 Ug' < Ui g'gY`Vgh``]Y[h'bYVYb'Y]bYa 'a U`Yf]gW\Yb'; Ya ØgY[UfhYb]b'XYf'BÀ\Y'Y]bYg'?]fgW\[UfhYbg"'; ÀghY'gc``hYb'Y]bYb'5i gZi [']bXUg'bU\Y'[Y`Y[YbY'8cfZ'6i bgW\chYb!GdU_YbVi f[']b'6YhfUW\hn]Y\Ybž'i a 'X]Y'hmd]gW\Y'b]YXYf`ÀbX]gW\Y'@UbXgW\UZh'i bX'Y]bYFY]\Y'j cb'A i gYYbž'; YgW\ÀZhYbž'A Àf_hYbž'FYghUi fUbhg'i bX'7UZÅgni 'gY\Yb"

6i $WXYbG]YX]YgYgBUhifUigBb6ibgWXchYb!gdU_YbVif[$

K]``_ca a Yb]b XYf; fUZgW UZh

 $8]YgYg'j ca '5i Yb`UbX']bgd]f]YfhY'BUhi f\Ui g']b'XYf'FY[]cb'A]X]!$

DmfÅbÅYgʻ]bʻ: fUb_fY]WXʻ]ghʻY]bYgʻi bgYfYfʻ`]YVghYbʻYWXhYbʻ<cVV]h! <Ài gYf"'Jc```a ÒV`]Yfh'a]hʻY]bYfʻØVYfXUWXhYbʻJ YfUbXUʻi bXʻY]bYa i a nÀi bhYbʻ; UfhYbžʻ\UhʻYgʻUi WXʻNi [Ub[ˈni a ˈi a `]Y[YbXYb; Y`ÀbXYʻi bXʻY]bYa 'Ub[fYbnYbXYbʻK U`Xžʻk cʻX]Yʻ; ÀghYʻY]b D]W_b]W_`Ua 'BUWXa]hhU[ˈa UWXYbʻ_ÒbbYb"

'U`h1 9]bYg'i bgYfYf'`]YVghYb'YW\hYb'<cVV]h!< $\dot{A}i$ gYf 2 BUhi f"<Ui g 9]bYg'i bgYfYf'`]YVghYb'YW\hYb'<cVV]h!< $\dot{A}i$ gYf

 $8]Y `A]X]! DmfÅbÅYg `g]bX `Y]bY `@UbXgWX UZh `Ui g `6Yf[Yb `i bX `K À`XYfbži bX `\]Yf `[]Vh `Yg `j]Y``ni `YbhXYW_Yb. `bUhØf`]WXY `< Ò\`YbžhfUX]h]cbY``Y `8 ÒfZYf `i bX `X]Y `6i f[Yb `XYf `? Uh\UfYf `j cb `A cbhgÅ[i fi bX `7 UfWUggcbbY" `8]Y `Gca a Yf `g]bX `[YZØ``h `a]h `8 cfZZYghYbžk À\fYbX `G_]ZU\fYf `]a `K]bhYf `]b `Y]b `') `_a `YbhZYfbhYg `G_][YV]Yh ZU\fYb `_ÒbbYb"$

6i WXYb G]Y X]YgYg B Uhi f\Ui g]b @]a VfUggUW

FØW_ni [gcfh]b XYb 6Yf[Yb

9]b'<c`nZYi Yfž'_f]ghU``_`UfYg`K UggYf`Ui g`Y][YbYf`E i Y``Y`i bX
B Uhi fghY]bk ÀbXY`a UWXYb`X]YgYg`B Uhi f\Ui g`ni a `dYfZY_hYb
F ØW_ni [gcfh"`8 Ug`]b`XYf`F Y[]cb`@Ub[i YXcWF ci gg]``cb`]b
: fUb_fY]WX`[Y`Y[YbY`<Ui g`k]fX`j cb`GcbbYb_c``Y_hcfYb`a]h`Ghfca
j Yfgcf[hii bX`j YfZØ[h'ØVYf`Y]bYb`[fc» Yb`HY]WX`Ui Z`XYa `8 UWX ž`XYf
Yg`]a `Gca a Yf`_Ø\``\À`h"

'U`h1 9]b'<c`nZYi Yf`Yfk UfhYh'G]Y']b'X]YgYa 'YW\hYb < cVV]h\Ui g 2 B Uhi f"< Ui g'9]b'< c`nZYi Yf`Yfk UfhYh'G]Y']b'X]YgYa YW\hYb'< cVV]h\Ui g

8]Y; ÀghY:_ÒbbYb:]a 'GWX UhhYb:Y]bYf:&\$\$:\\J\fY:U`hYb:GhY]bY]WXYZUi `YbnYbž'k À\fYbX:N]_UXYb:i bX; fY]Zj Ò[Y```Ui gWXYbž'X]Y:]b:XYb\c\Yb:VYk U`XYhYb:?`]ddYb:\]bhYf:XYa '< Ui g'b]ghYb"'Jcb:\]Yf:Ui g]ghYg'a Ò[`]WXž'Y]bYb:g]`VYfbYb:GhfY]ZYb:XYg'A]hhY`a YYfg'cXYf'X]YgWXbYYVYXYW_hYb;]dZY``XYf'DmfYbÀYb:ni 'Yf_YbbYb"

Ni 'XYb'5_h]j]hÀhYb']b'XYf'l a [YVi b['nÀ\'Yb'Gi fZYbž'GWk]a a Ybž

 $GY[Y^b'ibX'5b[Y^b'Ua'@UWXi'GU^U[ci'gck]Y'?UbiZU\fYb']a$ < $A^fUi'bz']a' < Yf]WcXYf']a'HUfb''$

6i WXYb G]Y X]YgYg B Uhi f\Ui g]b GU]bh df]j Uh

; UfhYbif`UiV

8]YgYg'BUhi f\Ui g'Ua 'K UggYf']gh'Y]bY': fYi XY'ZØf'; ÀfhbYf"'8Ug']bY]bYa 'Y\Ya U`][Yb'JYfgi Wkg[UfhYb']b'XYf'gØX\c``ÀbX]gWkYbGhUXh'6cg_ccd'[Y`Y[YbY'; Y`ÀbXY'k Uf'Y]bY': cfgWki b[gghUh]cb'ZØf6Ui a gWki `Yb'i bX'VY\YfVYf[h']a a Yf'bcWk'Y]bY'fY]WkY'5i gk U\`Ub'DZUbnYb'i bX'6Ài a Yb"

BUhi f"<Ui g'6cg_ccd]ghVYfØ\a h'ZØf'gY]bY'6Ui a gWki `Yb

8]Y'GhUXh'6cg_ccd'gY`Vgh']gh'VYfØ\a h'ZØf']\fY'6Ui a gW\i `Yb"'<]Yf VYZ]bXYb'g]W\'ØVYf'+\$\$'gck]Y'Y]b'6Ui a gW\i `a i gYi a ž'XUg'gY]bY `Ub[Y'i bX']``i ghfY'; YgW\]W\hY'U`g'6Ui a UbVUi [YV]Yh'nY][h"'8]Y ; ÀghY'_ÒbbYb'X]Y'GY\Ybgk ØfX][_Y]hYb'ni ': i » 'cXYf'a]h'XYa : U\ffUX'ØVYf'Y]bY'XYf'i bnÀ\`][Yb'_Ufh]YfhYb'Fci hYb'Yf_i bXYb"

6i $WXYbG]YX]YgYgBUhifUigBb6cg_ccd$

`c_U`Y'? Ògh`]W_Y]hYb

9]b'_b]ghYfbXYg': Yi Yfž'Y]bY']a '; UfhYb'b]ghYbXY'6`Ui a Y]gY'i bX']b XYf': YfbY'd`ÀhgW\Yfh'Y]b'6UW\"'K Ybb'X]Yg' \Rightarrow fY'JcfghY``i b['jcb Y]bYa 'dYfZY_hYb'5VYbX']ghž'XUbb'n]Y\Yb'G]Y'X]YgYg'BUhi f\Ui g']b XYf'FY[]cb'Hk YbhY']b'XYb'B]YXYf`UbXYb']b'6YhfUW\h"

BUhi f"<Ui g'9]b'_b]ghYfbXYg': Yi Yf'i bX'Y]b'd`ÀhgW\YfbXYf'6UW\k UfhYb'Ui Z'G]Y

9]b B Yhn j cb K Ub XYf! i b X F UXk Y [Yb V]YhYh Y]b Y a U Yf]g W Y A O [`]W _Y]h Ž X]Y @ Ab XYfY]Yb Ž K]Yg Yb ii b X K A`XYf j cb Hk YbhY ni Ybh XY W_Yb Ž UV Yf k]Y X]Y 9][Ybh Øa Yf g U [Yb Ž]gh XYf V Ygh Y K Y [Ž \Rightarrow f Yb 5 i Z Ybh \ U `h 'a]h Y]b Ya 'G b U \ W_]a 'b U \ Y [Y `Y [Yb Yb `F Ygh U if Ubh ni 'V Y []bb Yb Ž X Ug 'j cb '] \ f Ya 'B U W V U f b 'A U f W c c_ 'V Yhf]Y V Yb 'k]f X "; i h [Yb À \ fh 'i b X '[Yhf Àb_h _O bb Yb 'G] Y 'g d Un] Y f Yb '[Y \ Yb Ž Ub [Y`b c X Yf '[c `Z Yb Ž UV [Yf i b X Yh X i f W Y] b '6 U X ']b ' \Rightarrow f Ya 'd f]j Uh Yb K \]f`d c c `"

6i WXYb G]Y X]YgYg B Uhi f\Ui g]b 9bgWXXX

9]b < Ui W j cb F ca Ubh]_

±b'Y]bYf'a U`Yf]gWXYb'9W_Y'j cb'I]hk]^_']b'XYf'FY[]cb'BcfXVfUVUbh]b'XYb'B]YXYf`UbXYb'j YfV]bXYh'X]YgYg'BUhi f\Ui g'[Ya Øh`]WXYb?ca Zcfh'a]h'a cXYfbYa '?ca Zcfh'''9]b'_`UfYgž'a]b]a U`]gh]gWXYg 8Yg][b'a UI]a]Yfh'XYb'D`Uhn']a '±bYfYbž'k À\fYbX'Y]b'<c`ncZYbXfUi »Yb'Ui WX']b'_Ø\`Yb'BÀWXhYb'ZØf'`Ub[Y'fca Ubh]gWXY'5VYbXYgcf[h"

'U`h1 9]bY']XYU`Y'A]gW\ib['Uig'[Ya Øh`]W\Ya'?ca Zcfh'ibX a cXYfbYa'?ca Zcfh'ibX'a cXYfbYa'?ca Zcfh'ibX'a cXYfbYa'?ca Zcfh'ibX'a cXYfbYa'?ca Zcfh

9g'[]Vh'nU\`fY]W\Y'K UbXYf!'i bX'FUXk Y[Yž'X]Y'j ca '<Ui g
UV[Y\Ybž'i bX'a Y\fYfY'GY\Ybgk ØfX][_Y]hYb']b'XYf'BÀ\Yž
XUfi bhYf'XUg'XfUa Uh]gW\Y'K UggYfgW\`cgg'@cYj YghY]bž'X]Y
VYZYgh][hY'GhUXh'K ci Xf]W\Ya 'i bX'XYf'6]YgVcgW\!BUh]cbU`dUf_ž
Y]bYf'XYf'[fÒ» hYb'BUh]cbU`dUf_g'XYg'@UbXYg'i bX'Y]bYg'XYf
`YhnhYb'Ui g[YXY\bhYb'; YnY]hYb[YV]YhY'9i fcdUg'GØ» k UggYf!

 $: Yi \ WX \ h[\ YV]Y \ hY"$

6i WXYb^G]Y^X]YgYg^BUhi f\Ui g^]b^I]hk]^_

8 Y hU]`c

6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. 'Y]bY!fY]gY "XY