

7\i_\ib[5__`]a Uh]g]Yfi b[gk UbXYfi b[Yb ' 7\i_\i b['F]'/ Ua d/'=g`UbX'DYU_'6UgY 7Ua d

A ÒW hYb G]Y 7\i _\i b['F] c XYf XUg ⊕ `Ub X DYU_ 6 UgY 7 Ua d]b 5b[f]Z bY\a Yb 3 J]Y `Y] Wha ÒW hYb G]Y VY]XYg ni → fYf HfY__]b[fci hY ni `XYb XfY] `\c\Yb DÀggYb XYg 9j YfYgh \]bni ZØ[Yb " K]Y `Ui W]a a Yf Z X]YgYf @Y]h ZUXYb XYW_h G]Y `UV "FY]gYj Yfg]W Yfi b['ZØf' <]a U `UmU! HfY__]b[': Øf X]Y a Y]ghYb K Ub XYfYf Z X]Y `X]Y `8 fY] `DÀggY `VYk À `h][Yb a ÒW hYb Z ghY\h Y]bY FY]gYj Yfg]W Yfi b['[Ubn c VYb `Ui Z']\fYf `Df]cf]h AhYb `]ghY " → W _Ui ZY]a a Yf FY]gYj Yfg]W Yfi b[Yb `VY] K cf `XB ca UXg "Wca Z `XU ']W `[`Ui VYZ `XUgg `g]Y X]Y `VYghYb `ZØf 5 VYbhYi YffY]gYb XY `g]b X " `9g `i a ZUggh J Yfg]W Yfi b[ggW i hn ZØf a YX]n]b]gW Y J Yfgcf[i b[`]a '5 i g `Ub X Z *9j U_i]Yfi b[Z ; Yd A W_ i b X Y]bY FY]\Y j cb '5 VYbhYi YfgdcfhUfhYb i b X 5_h]j]h AhYb `k]Y `HfY__]b[`fUi W `]b `XYf < Ò\Y Ł" U `h1 / Ua d/, & \$ \hfY__]b[`fUi W `]b `XYf < Ò\Y Ł" \] \] a U `UmU/ Ua d/, & \$ \hfY__]b[`]a `b YdU` \] \]

A ÒW hYb G]Y 7\i _\i b[F] cXYf XUg ⊕ Ub X DYU_6UgY 7Ua d]b

5b[f]ZZ bY\a Yb 3 J]Y ``Y]W ha ÒW hYb G]Y VY]XYg ni =\fYf

HfY__]b[fci hY ni XYb XfY] \c\Yb DÀggYb XYg 9j YfYgh \]bni ZØ[Yb"

K]Y Ui W]a a Yfž X]YgYf @Y]hZUXYb XYW_h G]Y UV"

FY]gYj Yfg]WX Yfi b[ZØf <]a U`UmU!HfY__]b[

: Øf·X]Y·a Y]ghYb·K UbXYfYfž·X]Y·X]Y·8fY]·DÀggY·VYk À`h][Yba ÒWAhYbž·ghY\h·Y]bY·FY]gYj Yfg]WAYfi b['[Ubn·cVYb·Ui Z']\fYfDf]cf]hÀhYb`]ghY"

±WX _Ui ZY]a a Yf FY]gYj Yfg]WX Yfi b[Yb VY] K cf XBca UXg Wca ž XU

]WW [``Ui VYž'XUgg'g]Y'X]Y'VYghYb'ZØf'5VYbhYi YffY]gYbXY'g]bX"'9g i a ZUggh'J Yfg]WWYfi b[ggWWi hn'ZØf'a YX]n]b]gWWY'J Yfgcf[i b[']a 5i g`UbXž'9j U_i]Yfi b[ž'; YdÀW_'i bX'Y]bY'FY]\Y'j cb 5VYbhYi YfgdcfhUfhYb'i bX'5_h]j]hÀhYb'k]Y'HfY__]b['fUi WW']b'XYf <Ò\YŁ"

 $U^1 = hfY_{b}[^3]a bYdU^3 a U^1 = 2$

5bfY]gY bUW 7\i_\ib[

 $7 \le [3] ghY]b^{-}Y]bYgK c bXcfZUa FUbXY XYg^{-}X i _ i b[! HU^g" + YfghYf^@]b]Y X]YbhX]Y^{-}Ya Y]bXY XUni Z'8 fY]! < c Y!DAggY! HfY__Yf^{-} bX X]Y^{-}b][Yb^{-}ni _{-} Yfgcf[YbZ^{-}X]Y XYb^{-};]dZY^{-}XYg^{-}=a^{-}U^{-}HgY c XYf^{-}=q^{-}UbX^{-}DYU_{-}Yf_{-}^{-}]a a Yb^{-}a^{-}UW hYb^{-}$

 $\lain \lain \lai$

F9@5H98. 'H\fYY'DUggYg'DUW_'@]gh' 'K Ug'G]Y'ZØf'XYb'H\fYY'DUggYg'/'967'HfY_'VfUi W\Yb

5__`]a Uh]g]Yfi b[']b '7\i _\i b[

8 Yf??cb[a U'@U'DUgg'k]fX'j cb'K UbXYfYfb'U``[Ya Y]b'U`g'Y]bYf'XYf gWk]Yf][ghYb'i bX'UbghfYb[YbXghYb'DÀggY'Ui Z'XYf'hmd]gWkYb Fci hY'XYf'XfY]'\c\Yb'DÀggY'Ub[YgY\Yb"'9g'XUi Yfh'nk]gWkYb', i bX'%&'Ghi bXYb'i bX'Vf]b[h'G]Y'Ui Z'Y]bY'<Ò\Y'j cb'ØVYf')) \$\$ 'a "8U\Yf'k]fX'Xf]b[YbX'Ya dZc\`Ybž'g]Wk'bUWk'7\i _\i b['ni U__`]a Uh]q]YfYb'i bX'ni 'hfU]b]YfYb"

a 'A ca Ybh'g]bX'a Y]bY'; i]XYg'X]Y'Y]bn][Yb'EiY'`Yb'ZØf
cghYb`cgY' ₺\U`hYž'X]Y'cb`]bY'ZØf'XYb'8fY]!DÀggY!HfY'j YfZØ[VUf
g]bX"'K Ybb'G]Y'bUW\'k Y]hYfYb' ₺Zcfa Uh]cbYb'gi W\Ybž'_ÒbbYb'G]Y
Ui W\'Y]bY'?cd]Y'XYg'@cbY`m'D`UbYh 'HfY__]b[']b'h\Y'BYdU`
<]a U`UmU'6cc_'Yf\U`hYbž'XUg'ZØf'a]W\'gY\f'bØhn`]W\'k Uf"

K Y`WXYf'5VghYWXYf'gc``'[Ya UWXh'k YfXYb3'

K Ybb'G]Y'YXcW\'XYb'?cb[a U'@U'j Yfgi W\Ybž'k]fX'Xf]b[YbX Ya dZc\`Ybž'Y]bY'ni gÀhn`]W\Y'BUW\h']b'7\i _\i b['ni 'V`Y]VYbž XUa]h'G]Y'[YbØ[YbX'NY]h'\UVYbž'XUg'6Ug]g`U[Yf'XYg'-g`UbX'DYU_ni 'YffY]W\Ybž'cXYfž'k]Y']W\'Yg'[YhUb'\UVYž'VY]XY 5__`]a Uh]g]Yfi b[gk UbXYfi b[Yb'ØVYf'X]Y'nk Y]'HU[Y'ni UVgc`j]YfYb"

U`h1 9j YfYgh'H\fYY'<][\'DUggYg'HfY_ž'8fY]!DÀggY!: Ø\fYfž ?cb[a U'@U'DUgg 2

9g'[]Vh'_Y]bYb'9fgUhn'ZØf'Y]bY'[i hY'd\mg]gW\Y'?UfhYž'k Ybb'G]Y']a <]a U`UmU'g]bX"'8]Y'VYghY'?UfhY'ZØf'7\i _\i b['F]ž'9j YfYgh'6UgY 7Ua d'i bX'H\Y'H\fYY'DUggYg'HfY_'_Ubb'\]Yf'[Y_Ui Zh'k YfXYb"

$HfY_{_}b[$ $nia :]dZY`XYg.7\i_\ib[F]$

 $7 \le [T-1] \cdot [T-1] \cdot$

5 i g'X]YgYa '; fi bX'gc``hYb'K UbXYfYf'X]Y'K UbXYfi b['`Ub[gUa Ub[Y\Yb'i bX'Ub[Ya YggYbY'5__`]a Uh]g]Yfi b[gghfUhY[]Yb j YfZc`[Yb"

: Øf'j]Y`Y']gh'X]YgYf';]dZY``Y]bYf'XYf'gWkk]Yf][ghYb'HY]`Y']\fYf
9j YfYgh!K UbXYfi b["'8]Yg'`]Y[h'Ub'XYf'fY`Uh]j Yb'GhY]`\Y]h'i bX
XYa 'gWkbY``Yb'<Ò\YbUbgh]Y["'8UfØVYf'\]bUi g'k]fX'Yg
\`OWkghk U\fgWkY]b`]Wk'XUg'YfghY'A U`'gY]bž'XUgg'HfY__Yf'Y]bY'<Ò\Y
ØVYf')) \$\$ 'a 'YffY]WkYbž']bgVYgcbXYfY'8fY]!<c\Y!DAggY!HfY__Yfž
X]Y'[Y[Yb'XYb'I \fnY][Yfg]bb'k UbXYfb"
U`h1 6`]W_'j cb'7\i _\i b['F]']a '<]a U`UmU 2</pre>

K]Y ``Ub[Y XUi Yfh Ygž 7\i _\i b[F] ni k Ub X Yfb 3

: Øf`XYb`<]b!`i bX`FØW_k Y[`VYbÒh][Yb`G]Y`i b[YZÀ\f`(`Ghi bXYb HfY__]b[nY]h"`9g`k]fX`YXcW\`Ya dZc\`Ybž`g]W\`NY]h`ni `bY\a Ybž i a `DfcV`Ya Y`a]h`5A G`ni 'j Yfa Y]XYbž`XU`XYf`5i Zgh]Y[`gY\f`ghY]`]gh"

5 i Zb Y \ a Yb X Y g 7 \ i _ \ i b [F] H f U] `

*/ buw '*J\fYgnY]hi bX'HU[YgnY]h'\UVYb'G]Y'j]Y``Y]W\higW\cb
UbXYfY'K UbXYfYf'[YgY\Ybž'X]Y'XYb'UVZU``YbXYb'<Ø[Y``cVYf\U`V
j cb'7\i_\i b['\]bUi Z'cXYf'\]bi bhYf[Y[Ub[Yb'g]bX"'8Yf'K Y[']gh
j cb'YXYa 'Di b_h']b'XYf'GhUXh'Ui g'n]Ya `]W\'cZZYbg]W\h`]W\"

6Y[]bbYb'G]Y'a]h'Y]bYa'GdUn]Yf[Ub['nif'bcfXÒgh`]W\Yb'GY]hY'XYf GhUXh'i bX'bY\aYb'G]Y'XYb'KY['\]bhYf'XYf'AU_U`i'@cX[Y" JYf`UggYb'G]Y'X]Y'GhUXh'i bX'ØVYfei YfYb'G]Y'Y]bYb'_`Y]bYbž Z]Y»YbXYb'6UW\ž'VYj cf'G]Y'XYb'_i fj YbfY]W\Yb'GdUn]Yf[Ub['XYbYfghYb'ZY`g][Yb'<Ø[Y`\]bUi Z'VY[]bbYb"

8 Y f '5 i Zgh] Y [

 $B \ UWX \ Y]b \ Yf \ F \ Y] \ Yf \ cb \ GYfd \ Ybh]b \ Yb \ \ UVYb \ G]Y \ X]Y \ Gd]hn \ Yf \ ZY \ g][Yb \ 5VgWXb]hng \ jcf \ Y]b \ Ya \ gUb \ ZhYb \ zfi \ b \ XYb \ < \varnothing[Y``ni \ \rightarrow fYf \ FYWXhYb \ ib \ X'Y]b \ Ya \ gY \ fghY]``UV \ ZU``Yb \ XYb \ < \varnothing[Y`'jcf' \ b \ Yb \ YffY]WXh"$

K Ybb'G]Y'Ui Z'XYb';]dZY``XYg'7\i _\i b['F]'[Y`Ub[Yb'a ÒW\hYbž a ØggYb'G]Y'XYb'`Ub[Yb'i bX'a Ø\gUa Yb'5i Zgh]Y['XYb'ghY]`Yb < Ub['\]bUi Z'VY[]bbYb"

 $: \varnothing f^*X]Y \Upsilon b][Yb\check{z}^*X]Y^*g]WX^*] Yf^*5__^*]a Uh]g]Yfi b[^*b]WX h^*g]WX Yf^*g]bX\check{z}^*]gh^*Yg^*a \grave{O}[^*]WX\check{z}^*X]Y^*GY]hYbk UbXYfi b[^*Ui Z^*XYa ^*< \varnothing[Y^*ni ^+] fYf FYWX hYb^*ni ^*VYYbXYb''^K Ybb^*G]Y^*Ub^*X]Y^*Gd]hnY^*[Y^*Ub[Yb\check{z}^*k YfXYb K UbXYfYf^*) $$$ a ^*< \grave{O}Y^*YffY]WXYb\check{z}^*k Ug^*Z\varnothing f^*j]Y^*Y^*A YbgWXYb Ui gfY]WXYb^*_Ubb''$

Ui Z'XYa 'Ui g[YhfYhYbYb 'DZUX'XYb' < Ub['\]bUi Z'X]fY_h'ni a 5i gg]W\hgdi b_h'7\i _\i b['F]"

$7 i _i b[F] DYU_i bX 5i gg] Wh$

 $8] YgYf`5VgWX b] hh`]gh`Yhk Ug`_b] ZZ`][Yf`i bX`XYf`K] bX`\Yi `h`\] Yf VY_Ubbh`] WX`Xi fWX "`GcVU`X`G] Y``YXcWX`XYb`;] dZY``YffY] WX h`\UVYbž k YfXYb`G] Y`a]h`Y] bYa `Yhk Ug``c\bYbXYfYb`9f`YVb] g`XYf`[`Y] WXYb 5i gg] WX h`VY`c\bhž`XU`G] Y`[YfUXY`Y] bY`< Ò\Y`j cb`)) (*`a `YffY] WXh \AhhYb°$

'U`h1 9j YfYgh'H\fYY'<][\'DUggYg'HfY_ž'8fY]!DÀggY!: Ø\fYfž?cb[a U'@U'DUgg 2 U`h1 9j YfYgh'H\fYY'<][\'DUggYg'HfY_ž'8fY]!DÀggY!: Ø\fYfž?cb[a U'@U'DUgg 2 U`h1 7\i_i b['F=DYU_'5i gg]W\hgdi b_h 2

HfY__]b['ni a '=g`UbX'DYU_'f⊨a 'U'HgYŁ 6Ug]g`U[Yf

9]bY'k Y]hYfY'\U`VhÀ[][Y'5__`]a Uh]g]Yfi b[gk UbXYfi b['j cb 7\i_\i b['ZØ\fh'ni a '6Ug]g`U[Yf'=g`UbX'DYU_'f\alla 'U'HgY\z'XUg'Ui Z (-+\$'A'`]Y[h"

 $8 \ Ug^{\cdot}6 \ Ug]g^{\cdot}U[\ Yf^{\cdot}=g^{\cdot}Ub\ X^{\cdot}DYU_{-}^{\cdot}]gh^{\cdot}Y]b^{\cdot}Ui\ g[\ YnY]W\ bYhYf^{\cdot}5 \ VghYW\ Yf\check{z}$

U`h1 =g`UbX'DYU_'6UgY'7Ua d'HfUW_'5__`]a Uh]g]Yfi b[gk UbXYfi b[jcb'7\i_\i b['BYdU` 2

$=g'UbX'DYU_9IdYX]h]cb'HfY__]b[$

9gʻ]ghʻUi W. Y]bʻ[fc»Yfʻ: cfhgWkf]hhʻ[Y[YbØVYfʻXYaʻbcfa UʻYb<a>]a UʻUmU!HfY__]b["'K YbbʻG]YʻUʻgcʻXYbʻ[UbnYbʻK Y['ni a';]dZY`[Y\Ybʻa ÒWkhYbžʻa ØggYbʻG]YʻY]bYbʻ: Ø\fYfžʻY]bYʻ; YbY\a][ib[ibX`Y]bYbʻ5igZi[']bʻ?Uh\aUbXiʻUffUb[]YfYbʻcXYfʻg]WkʻY]bYf7fykʻ]bʻ7\i_\ib['UbgWk`]Y»Yb"

5V\c`i b['XYq'=q`UbX'DYU_'6UqY'7Ua d'HfU]`

8Yf =g`UbX DYU_'6UgY 7Ua d'HfU]`'VY[]bbh'Ua
[Y[YbØVYf`]Y[YbXYb'9bXY'jcb'7\i_\ib['nia'7\i_\ib['F]"
5bghUhh'Ua '9bXY'XYf'GhUXh'`]b_g'UVni V]Y[Ybž'V]Y[Yb'G]Y'fYW\hg
UV'i bX'ØVYfei YfYb'Y]bY'k Y]hYfY'dfcj]gcf]gW\Y'D`Ub_YbVfØW_Yž
X]Y'ØVYf'X]Y'6ÀW\Y'ZØ\fh"

9g'[]Vh'Y]bYb'_`UfYb'K Y[\check{z} 'i bX'Y'bUW\XYa \check{z} 'cV'= \check{g} `UbX'DYU_! GU]gcb']gh'cXYf'b]W\hž'[]Vh'Yg'a $\check{O}[`]$ W\Yfk Y]gY'Y]b][Y'HfÀ[Yf'i bX 9I dYX]h]cbgZØ\fYfž'X]Y'XYb'[`Y]W\Yb'?i fg'Y]bgW\`U[Yb"

 $8 \ Yf'DZUX'gWX`Ab[Y`h'g]WX'ni 'Y]bYa '_`Y]bYb`Yf\O\hYb'<\emptyset[Y``\]bUi Z i bX`ZØ\fh'XUbb`k Y]hYf'cVYb`\Yfi a ž'VYj cf'Yf']b`Y]b`Z`UWXYg'HU` UVZÂ``h"$

U`h1 =g`UbX'DYU_'6UgY'7Ua d'HfUW_'5___`]a Uh]g]Yfi b[gk UbXYfi b[jcb'7\i_\i b['BYdU' 2

8]Y = a'U'Hq c'5 dYbqYYb

BUWW i b[YZÀ\f'nk Y]Y]b\U`V'Ghi bXYb'HfY__]b['YffY]WWYb'G]Y'X]Y [fc»Yb'=a 'U'Hg\c'5`dYbgYYb"'K Ybb'G]Y'YXcWW'bi f'Ui Z'XYa 'DZUX $V`Y]VYb\check{z}'j$ YfdUggYb'G]Y'a O[`]WWYfk Y]gY'X]Y'VYghYb'5i gg]WWhYb"

9bh`Ub['XYg' =g`UbX'DYU_'6UgY'7Ua d'HfUW_g'gY\Yb'G]Y'Yf\Ò\hY <Ø[Y`ž'Xi fW\'X]Y'G]Y'\]bXi fW\'i bX'i a 'g]Y'\Yfi a [Y\Yb"'5b YXYa 'Di b_h']gh'Yg'a Ò[`]W\ž'Ui Z'X]Y'Gd]hnY'ni '_`YhhYfbž'i a 'Y]bYb VYggYfYb'6`]W_'Ui Z'X]Y'GYYb'Ui Z'XYf'UbXYfYb'GY]hY'ni '\UVYb"

: Øf a]W k Uf Yg Y]b gi ffYU Yf A ca Ybhž ØVYf X]Y GYYb ni gW Ui Yb i bX XYa a ÀW h][Yb?bUW_Yb i bX Ÿ W nYb XYg @\chgY!; YhgW Yfg ni "Ui gW Yb" 9g `Aggh G]Y k]f_`]W X]Y B ÒhY XYf A YbgW Yb gW AhnYbž X]Y]b X]YgYf YbgY]h][Yb @Ub XgW UZh `YVYb" U`h1 9j YfYgh H\fYY <][\DUggYg HfY_ž 8 fY]! DAggY!: Ø\fYfž?cb[a U @U DUgg 2

$=g`UbX`DYU_`6Ug]g`U[Yf$

? i fn `\]bhYf`XYb`GYYb`ZØ\fh`XYf`K Y[`Y]bY`k Y]hYfY`Ghi bXY``Ub[gUbZh`i bX`Z`UW\`\]bUVž`VYj cf`Yf`XUg`=g`UbX`DYU_`6UgY`7Ua d YffY]W\h"`8]Y`< \dot{O} \Y`VYhfÀ[h') \$, +A "

±b`'YXYa :: U```]gh'Yg`Y]bY`YfghUi b`]W\Y`@UbXgW\UZh'i bX`Y]bY dYfZY_hY`5__`]a Uh]g]Yfi b[gk UbXYfi b[ž'VYj cf'G]Y`XYb`?cb[a U`@U DUgg'j Yfgi W\Yb° U`h1 9j YfYgh'H\fYY`<][\'DUggYg`HfY_ž'8fY]!DÀggY!: Ø\fYfž?cb[a U`@U`DUgg 2

8fY] DÀggY. I bUV\Àb[][Yf HfY__]b[ZØ\fYf 9Vcc 'U`h1 8fY]'DÀggY.'I bUV\Àb[][Yf'HfY__]b[ZØ\fYf'j cb'C``m'; UgdUf
VY]'K Y'GYY_'HfUj Y` 2
8U'X]Y'\DhYfbYhj YfV]bXi b[']a '?\i a Vi 'VY[fYbnh']ghž'\UVY']W\
Y]bYb'i a ZUggYbXYb'HfY__]b[ZØ\fYf'ni 'XYb'8fY]'DÀggYb
[YgWXf]YVYbž'XYb'G]Y'U`g'Y6cc_'ni f'JYfk YbXi b['Ui Z'\D\fYa\
<UbXmž'HUV`Yh'cXYf'9!FYUXYf'\Yfi bhYf`UXYb'_ObbYb"</pre>

#W \\UVY X]YgYgYY6cc_ XYfnY]h'VY] 5a Uncb ni Y]bYa fYXi n]YfhYb
DfY]g'j cb k Yb][Yf'U`g'* '` [Y`]ghYh°

A 9-B 9 ? 5 A 9 F 5 I B 8 : C H C 5 I G F G H I B ;

- Gd]Y[Y``cgY'?Ua YfU.'7Ubcb'F)
- 8 fc \ b Y . '8 > A U | WDfc '&
- ' * \$!; fUX!5Wh]cb!?Ua YfU. '=bghU' * \$ 'CbY'L&
- @UbXgWXUZhqcVY h]j . '7 Ubcb'F: '%)!') 'a a 'Z#&", @
- 5``fci bX!CVY_h]j . '7Ubcb'F: '&(%\$) 'a a 'Z#(@
- HY`YcVY_h]| . 7 Ubcb F: '%\$\$) \$\$ a a 'Z#Z#(ž) + ž%'@
- @Ub[Y'5_h]cbgghUb[Y.'+bghU' *\$'+bj]g]V`Y'Dc`Y'fbi ``YhH]a YŁ
- @UbXgWXUZhg`]bgYbZ]`hYf.'< cmU'7]fW'`Uf'Dc`Uf]nYf
- ? Ua YfUfi W_gUW_. :: !Ghcd H]`cdU
- @|YV`]b[gZchcni VY\Of. DYU_8Yg][b'7Udhi fY'7`]d

9]bY'@]ghY'a Y]bYf'Ya dZc\`YbYb': chcUi gfØghi b['fM]bgW\`]Y»`]W\ k Ug']W\'VYbi hnY'i bX'k Ufi a Ł'Z]bXYgh'Xi ']b'a Y]bYa '@Y]hZUXYb'ZØf?Ua YfUUi gfØghi b['ZØf'X]Y'FY]gY"

K Ybb'G]Y'Y]bYg'XYf': chcg'Ui Z'X]YgYf'K YVg]hY'j Yfk YbXYb
a ÒW hYbž'VYgi W Yb'G]Y'V]hhY'a Y]bY'@]nYbn]Yfi b[ggY]hYž'i a
\YfUi gni Z]bXYbž'k]Y"'=\W 'j Yf_Ui ZY'Ui W 'dfcZYgg]cbY``Y
?i bghXfi W_Yž'VYgi W Yb'G]Y'a Y]bYb'Df]bh'GhcfY'cXYf'_cbhU_h]YfYb
G]Y'a]W 'X]fY_h'ZØf'Nc``UVnØ[Y'j cb'6]`XYfb'Ui Z'K Y'GYY_'HfUj Y`"
U`h1 2

п

6Ygi WXYb G]Y i bg Ui Z Y]bY!fY]gY "XY