

## : U]fm': U``g'7U]fbg' '8Yf'j YfghYW\_hY K UggYfZU```VY]'XYb'7fmghU`'7UgWUXYg

9]b 'gW\bY``Yf'K UggYfZU``ZØ\fYfž'XYf'=\bYb'\]`Zhž': U]fm': U``g'7U]fbgž XYb'j YfghYW\_hYb'K UggYfZU``'Ui Z'XYf'FØW\_gY]hY'j cb'7fmghU` 7UgWUXYgž'ni 'Z]bXYb"

: U]fm': U``g`7U]fbg`]gh'Y]b`\_`Y]bYg'j YfghYW\_hYg'¾ k Y`ž'bi f'Y]bYb \_i fnYb'%) !a ]bØh][Yb'GdUn]Yf[Ub['jcb'XYf'VY`]YVhYb'6UXYghY``Y 7fmghU``7UqWUXYg'YbhZYfbh"

A UbW\a U```c\bh'Yg`g]W\ž`UVgY]hg`XYf`Ui g[YhfYhYbYb`DZUXY`ni k UbXYfb"'8]Yg`[]`h`XYZ]b]h]j `ZØf`X]Y`: U]fm': U``gž`XU`X]Y`a Y]ghYb 6Ygi W\Yf`XYf`7fmghU``7UgWUXYg`b]W\h'Y]ba U``k ]ggYb`k ØfXYbž XUgg`g]Y`YI]gh]YfYb°`A Y]bYf`A Y]bi b[`bUW\`]gh'Yg`Y]b`gW\ÒbYfYf K UggYfZU``ž`Y]b[YVYhhYh`]b`Y]bY`\_`Y]bY`FY[Ybk U`XYW\_Yž`\_ca d`Yhh a ]h`\Àb[YbXYb`FYVYb`i bX`Y]bYa `\_f]ghU``\_`UfYb K UggYfZU``VYW\_Yb"

±b 'X |Y gY a '@Y |hZUXY b 'hY | 'Y '|WX ' → bY b 'U``Y g 'a |hž 'k Ug 'G |Y 'ØVY f 'XY b

6Ygi W\'j cb': U]fm': U``g'k ]ggYb'a ØggYbž'Y]bgW\`]Y $\gg$ `]W\'XYf'5bfY]gY i bX'k Ug'G]Y'bUW\' $\Rightarrow$ fYf'5b\_i bZh'Yfk UfhYh"

K Ybb'G]Y'a Y]bYb'6'c['ni a 'YfghYb'A U```YgYbž'k]ggYb'G]Y
j ]Y``Y]W\h'b]W\hž'XUgg']W\'<i bXYfhY'j cb'Yd]gW\Yb
GY\Ybgk ØfX][\_Y]hYb'i bX'5\_h]j ]h\hYb']b'7U]fbg'Xc\_i a Ybh]Yfh
\UVY"BUW\XYa 'G]Y'g]W\'X]Y'7fmghU``7UgWUXYg'Ub[YgY\Yb'\UVYb\z
Yf\_i bXYb'G]Y'XcW\'X]Y'Yd]gW\Yb'7U]fbg!K UggYfZ\\``Y'cXYf
K UbXYfk Y[Y'cXYf'gY\Yb'G]Y'g]W\'X]YgY'@]ghY'Ub\z'i a 'ni 'gY\Yb\z
k c'G]Y'ØVYfbUW\hYb'\_\ObbYb"

U`h1 :  $5 \neq M$ :  $5 \neq M$ :  $5 \neq M$ GH5@75G7589G75 $\neq$ BG 2: U]fm: U``g

J 9FK 5B8H9": , < F9F. "GhcbYm"7fYY\_": U``g'p"?f]ghU``\_Ug\_UXYb"

#### 6YghY K UggYfZU``hci fYb ]b 7U]fbg

6Yj cf'k ]f'i bg'a ]h'X]YgYa 'Yd]gW\Yb'j YfghYW\_hYb'≯k Y`'Y]bYg K UggYfZU``g'VYZUggYbž'a ÒW\hY']W\'a ]h'→bYb'X]Y'VYghYb'7U]fbg! K UggYfZU``hci fYb'hY]`Yb"

Hci f	kUgʻ]ghʻ]bVY[f]ZZYb
K UggYfZU``!žK ]`Xh]Yf!'i bX	ægYd∖]bY∵U``gžA]``UUA]``UUž
FY[Ybk U`Xhcif	8]bbYf': U``gž'GW\bUVY`h]Yf'VY]
	M b[ UVi ffUž'@U_Y '9UW Ua
; Øbgh][ghY'5h\Yfhcb'HUV`Y`UbXg	ægYd∖]bY∵U``gž'A]``UU'A]``UUž
K UggYfZU``hcif	8]bbYf": U``gž
	Jcf\Ub[ZY][YbVUi a
5h\Yfhcb'HUV`Y`UbXg	?f]ghU``_Ug_UXYbž'≈gYd∖]bY
K UggYfZÀ``Y'i bX'HU[Yghcif	: U``gž'A ]``UU'A ]``UUž'6UV]bXU
Xi fWX XYb FY[Ybk UX UV	6ci `XYfgž'@U_Y '9UW\Ua ž
7U]fbg	

'U'h1 A =@55'A =@55'K 5GG9F: 5@ 2

#### K ]Y \_ca a ha Ub bUW : U]fm: U``g 7U]fbg3

I a 'ni 'XYb': U]fm': U``g'ni '[Y`Ub[Ybž'a ØggYb'G]Y'ni 'XYb'7fmghU` 7UqWUXYg'ZU\fYbž'X]Y'Yhk U'&) 'A ]bi hYb'j cb'XYf'GhUXh'7U]fbg

 $\label{thm:continuous} $$ YbhZYfbh`]Y[Yb"@Y]XYf'[]Vh'Yg'_Y]bY'OZZYbh`]WXYb'JYf_Y\fga]hhY`$ ni `XYb'7UgWUXYgž'gcXUgg'G]Y'$\Rightarrow$f'Y][YbYg'HfUbgdcfha]hhY`$ VYbOh][Yb"$$ 

8 Yf'DUf\_d`Uhn'ZØf'X]Y'7 fmghU`'7 UgWUXYg'VYZ]bXYh'g]W\'Ua '9bXY
XYf'FYX`mbW\'\=\bhU\_Y'FcUX"'<]Yf'a ØggYb'G]Y'\]bZU\fYb'i bX'\=\f
5i hc'UVghY``Ybž'VYj cf'G]Y'XYb'[Y\Y]a Yb'DZUX'ni 'XYb': U]fm: U``g
YffY]W\Yb"'8]Y'[YbUi Y'@U[Y'XYg'DUf\_d`UhnYg'\UVY']W\'Ui Z'XYf
? UfhY'i bhYb'a Uf\_]Yfh"

K Ybb'G]Y'\_Y]b'Y][YbYg'5i hc'\UVYbž'XUbb'Ya dZY\`Y']W\'\⇒\bYbž Y]bY'7Ubmcb]b[!Hci f'UV'7U]fbg'ni 'Vi W\Yb"'8Yf'Zc`[YbXY']gh'Y]bYg XYf'VYghYb'7Ubmcb]b[!9f`YVb]ggY']a '\c\Yb'BcfXYb'j cb Ei YYbg`UbX'i bX'ZØ\fh'XYb' 7fmghU`'7Ubmcb '\]bUi Zž'[Ubn']b'XYf BÀ\Y'XYf'?Ug\_UXYb'i bX':U]fm':U``q"

DfØZYb G]Y XYb VYghYb DfY]g. 7U]fbg 7UbmcbYYf]b[ 5Xj Ybhi fY

'U`h1 : 5 ≠ M: 5@@G'7F MGH5@'75G7589G'65F CB': 5@@G B5H+CB5@D5F?'75 ≠ BG 2GWk]a a Yb'VY]': U]fm': U``g'VY] a Y]bYa 'YfghYb'6Ygi W

#### $8]Y'jYfghYW_hY'GdifiniXYb:U]fm:U``g'Z]bXYb$

 $8 \ Yf'K \ Y[\ 'ni \ 'XYb': \ U]fm': \ U``g'VYZ]b \ XYh'g]W. \ 'Ui \ Z'XYf'`]b \_Yb'GY]hY'XYg \\ DUf \_d`UhnYg'fk \ Ybb'G]Y'XYb'DUf \_d`Uhn'VYhfYhYb'i \ bX']b'F]W. \ hi \ b[ XYg'7fmghU``7UgWUXYg!K \ UbXYfk \ Y[\ g'V`]W_Yb'.''$ 

9g'[]Vh'\_Y]bY'GW\]`XYf'cXYf'A Uf\_]Yfi b[Ybž'X]Y'XYb'K Y[UbnY][Ybž'UVYf'G]Y'gY\Yb'X]Y'@]W\hi b['bYVYb'Y]bYa 'K UfbgW\]`X ZØf'ghYW\YbXY'DZUbnYb"

 $: c`[Yb`G]Y`X]YgYa `DZUX`b]WXh``Ab[Yf`U`g`%$`A]bi hYb`Xi fWX`XYb X]WXhYb`hfcd]gWXYb`FY[Ybk U`Xž`V]g`G]Y`XYb`6UWX`YffY]WXYbž`k c XYf`DZUX`bUWX``]b_g`UVV]Y[h"`BY\aYb`G]Y`b]WXh`XYb`hfØ[Yf]gWXYb DZUXž`XYf`ZUgh`gYb_fYWXh`Y]bYb`<<math>\emptyset[Y``\]bUi ZZ\emptyset\fh"`6`Y]VYb`G]Y ghUhhXYggYb`]b`XYf`BÀ\Y`XYg`6UWXYg`i bX`[Y\Yb`G]Y`k Y]hYfž`V]g G]Y`Y]bY`_`Y]bY`@]WXhi b[`YffY]WXYb"$ 

< ]Yf'[ ]Vh'Yg'Y]bYb'Z'UWXYb'Dcc``i bX'X]fY\_h'XUfØVYf'XYb
j YfghYW\_hYb'K UggYfZU``"</pre>

U`h1 :  $5 \neq M$ : 5@G HF 57? 69 = 7 F MGH5@ 75G7589G

75 ≠ BG 2: U]fm': U``g'K UbXYfk Y[

#### : U]fm: U``g'7U]fbg' K Ug'G]Y'Yfk UfhYh'

9gʻgh]a a hžʻ: U]fmʻ: U``gʻ]ghʻb]Wk hʻXYfʻVYY]bXfi W\_YbXghYʻK UggYfZU`` ]bʻ7U]fbgʻ]bʻ6Yni [ʻUi Zʻ; fÒ» YʻcXYfʻ?fUZh"·9gʻ]ghʻYXcWkʻY]bʻgY\f a U`Yf]gWkYfʻK UggYfZU``žʻXYfʻ]bʻX]Yʻ?`]ddYʻ[Y\Ui Ybʻ]ghʻi bXʻ]b Y]bYbʻgWk a U`YbžʻUVYfʻh]YZV`Ui YbʻDcc``a ØbXYh" U`h1 7U]fbgʻ: U]fmʻ: U``g 2 9gʻ[]VhʻUi WkʻY]bYbʻ[fc» YbžʻX]W\_Ybʻ6Ui a ghUa a žʻXYfʻX]Y Ghfca gWkbY``Ybʻ\Yfi bhYf[Yf]ggYbʻi bXʻg]WkʻdYfZY\_hʻ]bʻXYb K UggYfZU```[Y\_`Ya a hʻni ʻ\UVYbʻgWkY]bh"·8]Yʻ9]b\Y]a ]gWkYb\UVYbʻY]b][YʻGhi ZYbʻ]bʻX]Yʻfi hgWk][YʻF]bXYʻ[Y\Ui YbžʻXUa ]hʻG]Y Yhk Ugʻ\Ò\Yfʻ\_`YhhYfbʻi bX`XYbʻK UggYfZU```VYk i bXYfbʻ\_ObbYb"

: U]fm: U``g`]gh'Y]bYf'a Y]bYf'@]YV`]b[gcfhY']b'7U]fbgži a 'UVgY]hg XYf'A YbgWXYba UggYb'ni 'YbhgdUbbYb"'8Yf'GdUn]Yf[Ub[']gh'gY\f\_ifn'i bX'Y]bZUWXž'k Ug']\b'ni 'Y]bYa 'XYf'VYghYb'CfhY']b'XYf'GhUXh a UWXhži a 'g]WX']b'XYf'\Y]»Yb'hfcd]gWXYb'<]hnY'UVni \_Ø\`Yb" U`h1 :  $5 \neq M$ : 5 = 675 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 656 = 65

### $7U]fbgTbhYf_ibZhgZ\emptyset\fYf^Kc^{\circ}VYfbUWhYb$

D`UbYb`G]Y`Y]bY`FY]gY`bUW\`7U]fbg`i bX`ZfU[Yb`g]W\ž`k c`G]Y`g]W\
b]YXYf`UggYb`gc``Yb3`I bhYb`\UVY`]W\`Y]b][Y`XYf`VYghYb`Cdh]cbYb
]b`XYf`BÀ\Y`j cb`7U]fbg`9gd`UbUXY`Ui Z[Y`]ghYh"`=W\`\UVY`Ui W\
Y]bYb`i a ZUggYbXYb`I bhYf\_i bZhgZØ\fYf`ZØf`7U]fbg`gck]Y`Y]bYb
i a ZUggYbXYb`I bhYf\_i bZhgZØ\fYf`ZØf`Dcfh`8ci [`Ug`[YgW\f]YVYb"

7fmghU`Vfcc\_'F]`Ym' 'K Ybb'G]Y'bUW\'@ Iig'ibX'@U[Y'gi W\Ybž']gh XUg'7fmghU`fVcc\_'F]`Ym'ibgW\`U[VUf"'8]Yg']gh'XUg'VYghY')!GhYfbY!FYgcfh']b'7U]fbg'ibX'V]YhYh'UVgc`ih'U``Ygž'kUg'G]Y'jcb'Y]bYa7fmghU`Vfcc\_!FYgcfh'YfkUfhYb"

H]dd.; Y\Yb'G]Y'Ui Z'YXYb': U```ni 'FcWWc g'6Uf'Ui Z'XYf'Gd]hnY'XYg 7fmghU`Vfcc\_'F]`Ymž'i a 'Y]bYb'[fc»Ufh][Yb'6`]W\_'ØVYf'7U]fbg'ni [Yb]Y»Yb°

U`h1 A ÀXWXYb`VY]a 'GcbbYbVUXYb']a '7fmghU`Vfcc\_'F]`Ym`FYgcfh]b'7U]fbg 2

FmX[Yg'FYgcfh' '8]YgYg'<chY`'V]YhYh'Y]b'\YfjcffU[YbXYg'DfY]g!
@Y]ghib[g!JYf\À`hb]g'ZØf'Y]bYb'Yd]gW\Yb'5iZYbh\U`h'ni'Y]bYa
UbghÀbX][Yb'DfY]g\*'8]Y'@U[Y']gh'dYfZY\_hž']b'XYf'BÀ\Y'XYf
9gd`UbUXY'ibX'jYfZØ[h'ØVYf'XfY]'Gk]aa]b[dcc`gž'nkY]
HYbb]gd`ÀhnYž'Y]bYb'K\]f`dcc`ž'Y]b':]hbYggWYbhYf'ibX'gc[Uf'Y]b
dfY]q[Y\_fÒbhYq'FYghUifUbh"

U`h1 FMB; 9G'9GD@5B589'F9GCFH'₺'75∓BGž'K C'IBH9F?IB:H NI'6@9₺9B2

: ]hnfcm=g`UbX`FYgcfh` 'K Ybb`G]Y``]YVYf`Ui Z`Y]bYf`XYf`a U`Yf]gWXYb
=bgY`b`XYg`; fYUh`6Uff]Yf`FYYZ'ØVYfbUWXhYb`a ÒWXhYbž'g]bX`G]Y`]a
: ]hnfcm=g`UbX`FYgcfh`[YbUi `f]WXh]["`K À\fYbX`G]Y`\]Yf`Y]bYb
HU[YgUi gZi [`i bhYfbY\a Yb`\_ÒbbYbž'Ya dZY\`Y`]WX`Xf]b[YbXž'X]Y
BUWXh`ni `j YfVf]b[Ybž'XUa ]h`G]Y`a Y\f`NY]h`\UVYbž'XUg`F]ZZž'X]Y
?cfU``YbghfÀbXY`i bX`X]Y`K UbXYfk Y[Y`]a `FY[Ybk U`X`ni
Yf i bXYb"

U'h1 :  $\pm$ NFCM $\pm$ @5B8'F9GCFH 2

; ]``][ Ub g'6UW\_dUW\_Yfg' '8]Y'VYghY'6UW\_dUW\_Yf!Cdh]cb'ZØf'Y]bY hc``Y'5ha cgd\AfY']b'7U]fbg']gh; ]``][ Ub g'6UW\_dUW\_Yfg"'8Ug'`]Y[ h XUfUbž'XUgg'X]YgYg'Y]bYb'Gk ]a a ]b[ dcc`ž'Y]bY'?`]a UUb`U[ Yž'Y]bY \_ca d`YhhY'?ØW\Y'i bX'Y]bYb'[ fc»Ufh][ Yb'; Ya Y]bgW\UZhgVYfY]W\\Uhži a 'UbXYfY'FY]gYbXY'ni 'hfYZZYb"

'U'h1 ; =@=; 5BG'657?D57?9F'F9GCFH2

"U"h1 :  $5 \neq M$ :  $5 = G^75 \neq BG$  2

# A Y\f'K UggYfZÀ``Yž'K UbXYfi b[Yb'i bX 5VYbhYi Yf']b'7U]fbg

⇒W. `\cZZYž'XUgg'X]YgY'?i fnUb`Y]hi b[ 'ni a '6Ygi W. 'XYf': U]fm': U``g']b
7U]fbgž'Ei YYbg`UbXž'ZØf'G]Y'\]`ZfY]W. 'k Uf"'K Ybb'⇒\bYb'X]YgYf'Cfh
[YZU``Yb'\Uhž'gW.Ui Yb'G]Y'g]W. 'i bVYX]b[h'Y]b][Y'a Y]bYf'UbXYfYb
5fh]\_Y``ZØf'XYb'\c\Yb'BcfXYb'Ub"'⇒W. '\UVY'i]Y``NY]h'XUa ]h

j YfVfUW\hž\X]Y\VYghYb\5VYbhYi\Yf\ni\\Xc\_i\ a\Ybh]YfYb\ž\i\ a\\g]Y\\alpha\]\  $\Rightarrow$  bYb\\ni\\\h\Y\)\\"

'U`h1 2

п

#### 8 Y hU1`c

6Ygi WXYb'G]Y'i bg'Ui Z.'Y]bY!fY]gY"XY