

5VgY]hg XYf K Ub X. 8 Yb E i YYbghck b! ? `YhhYfghY]['Yf_`]a a Yb

8 Yf'Ei YYbghck b!?`YhhYfghY]['Yfa Ò[`]WkhYg'?`YhhYfYfbž UbgcbghYb'i bdUgg]YfVUfY'?`]ddYb'ni 'Yf_`]a a Yb'i bX'ni Yf_`]a a YbžX]Y'g]Wk'\cWk'ØVYf'XYf' 5VYbhYi Yf\Ui dhghUXh XYf'K Y'h 'Yf\YVYb"'9g']gh'gY'hYbž'XUgg'Y]b'6Ygi WkYf Ei YYbghck b']b'BYi gYY`UbX'j Yf`Àgghž'c\bY'Yhk Ug'dfcV]Yfh ni '\UVYbž'XUg'gY]bYb'Di `g'\Ò\Yf'gWk`U[Yb'`Àggh"'K]f Vygi WkhYb'X]Y' 5VYbhYi Yf\Ui dhghUXh'XYf'K Y'h ']a 'FU\a Yb Y]bYf'Hci f'j cb'; '5Xj Ybhi fYg'Ui Z'XYf'GØX]bgY`ii bX'k c``hYb i bVYX]b[h'X]Y'9bnm_`cdÀX]Y'XYf'GhUXh'a]h UXfYbU`]b[Y`UXYbYb'5_h]j]hÀhYb'Ui gdfcV]YfYb"'BYi gYY`UbX k i fXY'j cb'@cbY`m'D`UbYh'U`g'Y]bYg'XYf'VYghYb'@AbXYf'ZØf Y]bYb'6Ygi Wk']a '\$J\f'&\$%, 'VYnY]WkbYh'i bX'V]YhYh'ZUgh YbX`cgY'A Ò[`]Wk_Y]hYb'ZØf'5VYbhYi Yf'i gh][Yž'XUfi bhYf 6i b[ml*a d]b[ž'7Ubmcb!Gk]b[]b[z': U``gWk]fa gdf]b[Yb'i bX G\chcj Yf!*YhVcchZU\fYb'/ Ua d\Y'`]d/

8 Yf'Ei YYbghck b!?`YhhYfghY]['Yfa Ò[`]W\h'Yg'?`YhhYfYfbž UbgcbghYb'i bdUgg]YfVUfY'?`]ddYb'ni 'Yf_`]a a Yb'i bX'ni Yf_`]a a Ybž'X]Y'g]W\'\cW\'ØVYf'XYf' 5VYbhYi Yf\Ui dhghUXh'XYf K Y`h 'Yf\YVYb"

9gʻ]ghʻgY`hYbž'XUggʻY]bʻ6Ygi WXYfʻE i YYbghck bʻ]bʻBYi gYY`UbX j Yf`Àgghžʻc\bYʻYhk UgʻdfcV]Yfhʻni ʻ\UVYbž'XUgʻgY]bYbʻDi `gʻ\Ò\Yf gWX`U[Ybʻ`Àggh''K]fʻVYgi WXhYbʻX]Yʻ 5VYbhYi Yf\Ui dhghUXhʻXYf K Y`hʻ]a ʻFU\a YbʻY]bYfʻHci fʻj cbʻ; ʻ5Xj Ybhi fYgʻUi ZʻXYfʻGØX]bgY` i bX`k c``hYbʻi bVYX]b[hʻX]Yʻ9bnm_`cdÀX]YʻXYfʻGhUXhʻa]h UXfYbU`]b[Y`UXYbYbʻ5_h]j]hÀhYbʻUi gdfcV]YfYb''

BYi gYY`UbX`k i fXY`j cb`@cbY`m`D`UbYh`U`g`Y]bYg`XYf`VYghYb @ÀbXYf`ZØf`Y]bYb`6Ygi W\`]a `\\\f`&\%, `VYnY]W\bYh`i bX`V]YhYh`ZUgh YbX`cqY`A Ò[`]W_Y]hYb`ZØf`5VYbhYi Yf`i qh][Yž`XUfi bhYf`6i b[ml \Rightarrow a d]b[\check{z} 7 Ubmcb!Gk]b[]b[\check{z} 1: U``gWk]fa gdf]b[Yb'i bX'G\chcj Yf! \Rightarrow hVcchZU\fYb'ffU \check{z} 1k]f'\UhhYb'Ui Wk'_Y]bY'5\bi b[\dot{z} 1"

K À\fYbX'?]U``cgnc[žia 'XYb'%' ('a '\c\Yb'BYj]g'6ib[m'ni gdf]b[Ybž'dfcV]YfhY']W\'`]YVYf'Yhk Ug'Ui gž'VY]'XYa 'Yg'b]W\h'i a XYb'ZfY]Yb': U``'[]b['''[Ubn']a'; Y[YbhY]`"'\=\W\'YbhgW\]YX'a]W\'ZØf XYb'Ei YYbghck b!?`YhhYfghY][ž'Y]bY'FY]\Y'j cb'?`YhhYffci hYbž'X]Y <i bXYfhY'j cb'A YhYfb'X]Y'?`]ddYb'ØVYf'Ei YYbghck b'Ui Z BYi gYY`UbXg'YfghYa'?`YhhYfghY]['\]bUi ZghY][Yb"

K Ug]gh? `YhhYfghY][3

 $\label{eq:continuity} $$J_U: YffUhUž``UhY]b]gWX`ZØf` 9]gYbk Y[\check{z}']gh'Y]bY'6fØW_Y'nk]gWXYb?fUIY`b'i bX'?`YhhYfb"'9g']gh'YI dcb]YfhYf'U`g'?`YhhYfb'a]h$$VYggYfYbž'i b[Y\]bXYfhYb'5i gg]WXhYb"'=a '; Y[YbgUhn'ni a : Y`g_`YhhYfb'YfZcfXYfh'Yg'YXcWX'bi f'gY\f'k Yb]['5i gfØghi b['cXYf_ca d`]n]YfhY'GY]`UfVY]hYb"$

8U'X]Y'Fci hY'Ui »YfXYa 'a Uf_]Yfh']ghž'k]fX'Y]b'; fc»hY]`'XYg GhfYggYg'j Yfa]YXYbž'XYf'a]h'XYf'DfcV'Ya 'Ògi b['VY]a '6ci `XYfb Y]b\Yf[Y\h''? `YhhYfghY][Y'Yfa Ò[`]W\Yb'DYfgcbYb'a]h Y]b[YgW\fÀb_hYf'cXYf'_Y]bYf'? `YhhYfYfZU\fi b['XYb'Ni [Ub['i bX ;]dZY`'ni 'gcbgh'i bYffY]W\VUfYb'< Ò\Yb"

 $?`YhhYfghY][Y'g]bX'jc``ghÅbX]['ibX'XUiYf\UZh'a]h'GhY][\]`ZYbUig[YghUhhYhž'ia'XYb'5iZgh]Y['ni'Yf`Y]W\hYfb'ibX'X]Y'G]W\Yf\Y]hni'Yf\O\Yb'''9]bY'FY]\Y'jcb''9]gYbgdfcggYb'fGhYadY`bŁ'g]bX'UbXYf':Y`gkUbX'VYZYgh][h'ibX'V]YhYb'Y]bYb'`Y]hYfUfh][Yb'Ni[Ub[nia'?`YhhYfb''$

'U`h1 Ei YYbghck b'J]U': YffUhU'9]gYbgdfcggYb 2 HfUi a nY]h'9]bY FY]\Y'j cb'9]gYbgdfcggYb'g]bX'Ub'XYf': Y`gk UbX'VYZYgh][h

BYVYb'XYf'HfUggY'j Yf`Ài Zh'Ui W\'Y]b'GhU\`gY]`ž'XUg'dYf]cX]gW\'fU``YdUUf'A YhYfŁ'Ua': Y`g'VYZYgh][h'k]fX"'A UbW\a U`'_ÒbbYbni gÀhn`]W\Y'?`YhhYf\]`ZYb'k]Y'<Yf]b[Yž'[YgW\b]hnhY'Ghi ZYb'i bXgc[Uf'@Y]hYfb'cXYf'6fØW_Yb'j Yfk YbXYh'k YfXYb"

J]U:: YffUhU:[Y\h:Ui Z:XUg:%- ":IJ\f\i bXYfh:ni fØW_ž:U`g:6Ui Yfb:]b XYb:Yi fcdÀ]gWXYb:5`dYb:X]YgY:HYWXb]_:bi hnhYbž:i a :8 ÒfZYf:a]h \Ò\YfYb:K Y]XYb:ni :j YfV]bXYb":9g:k i fXY:XUbb:j Yfk YbXYhž:i a XUg:K UWXghi a :XYf:U`d]bYb:9fZcfgWXi b[:i bX:XYg:Hci f]ga i g:ni i bhYfghØhnYbž:X]Y:g]WX:UV:A]hhY:XYg:IJ\f\i bXYfhg:Ybhk]W_Y`hYb"

EiYYbghck b!?`YhhYfghY][

; ` $OW_$ `]WXYfk Y]gY'k YfXYb'? `YhhYfghY][Y'\Yi hY'k Y`hk Y]h'bi f'bcWX ni '9f\c`i b[gnk YW_Yb'[Ybi hnh"'8Yf'Ei YYbghck b!? `YhhYfghY][i a ZUggh'%&'Fci hYb'a]h'OVYf'' \$\$'<O\Yba YhYfb'i bX']gh'bi f'Y]bY _i fnY'5i hcZU\fh'j ca 'NYbhfi a 'j cb'Ei YYbghck b'YbhZYfbh"

 $8]Y ??`] ddYb ii a ZUggYb Y]bY FY]\Y ij cb Fci hYb ia]h$ $i bhYfgWX]YX`]WXYb GWXk]Yf][_Y]hg[fUXYbž Y bUWXXYa ž k]Y ig]WXYfG]Y ig]bX"'6Y[]bbYbX Ua :: i <math> > Y$]bYg [fUgVYk UWXgYbYb i $< \emptyset [Y `g UfVY]hYb ig]WX iØVYf '%"$$$ GdfcggYb ii bX i) $$ ia i? UVY`X]Y UWXh : Y`gk ÅbXY \]bUi Z''$

'U`h1 EiYYbghckb'?`YhhYfghY]['EiYYbghckb 2 HfUi a nY]h'8]Y
?`]ddYb'V]YhYb'Y]bY'\Yfj cffU[YbXY'5i gg]W\h'ØVYf'EiYYbghckb

9g'k Uf'gdÀh']b'XYf'GU]gcbž'U`g'k]f'Xcfh'k UfYbž'U`gc'k UfYb'bi f'j]Yf UbXYfY'; ÀghY'Ui Z'a Y]bYf'6YghY][ib["'8U'k]f'ibg'U``Y'n]Ya`]Wk g]WkYf'ZØ\`hYb'fhfchn'XYg'b]Wkh'dYfZY_hYb'K YhhYfgŁž'YbhgWk]YXYb k]f'ibg'ZØf'X]Y'Ubgdfi Wkgj c``YfYb'FcihYb'ibX'hYWkb]gWkYb: UWYhhYb'XYf'UWkh'?`]ddYbUVgWkb]hhY"

 $\begin{tabular}{ll} K & $[f'VY[UbbYb'Ub'XYf'6Ug]g'ibX'kifXYb'gcZcfh'jcb'Y]bYf \\ VfUigZcfXYfbXYb'?`YhhYfY]'ØVYffUgWkh''9g'XUiYfhY'b]Wkh'`Ub[Yž$

V]g'X]Y'; YfÀi gW\Y'XYf'GhUXh'i bhYf']\bYb'ni 'j Yfghi a a Yb VY[UbbYb"

VYffUgWXYbXYfk Y]gY 'k UfYb 'X]Y 'YfghYb '? `]ddYbž'X]Y 'U`g' I bhYfY

VYnY]WXbYh'k YfXYbž'gWXk]Yf][Yf'ni 'Yf_`]a a Yb 'U`g'X]Y ' CVYfYb "

8]Y 'F ci hY 'VY]b\U`hYhY 'Y]b][Y 'gWXk]bXY`YffY[YbXY', VYf\Àb[Yž'k c
k]f'[Ynk i b[Yb 'k UfYbž'XYf'G]WXYf\Y]hgUi gfØghi b['ni 'j YfhfUi Yb
i bX i bg'j cb 'XYf': Y`gk UbX'ni 'Y\bYb"

9]b A]h[`]YX i bgYfYf'; fi ddY ' 'Y]b '`Ub['À\f][Yf GWk]bXY`_fUb_Yf VYgW\`cggžXUgg'Yf'[Ybi ['\UhhYž'i bX'XfY\hY'g]W\ 'a]h'Y]bYa : Ø\fYf'i a ž'XYf']\b 'ni fØW_']b 'X]Y 'GhUXh'VY[`Y]hYhY"'9g']gh Yfk À\bYbgk Yfhž'XUgg'? `YhhYfghY][Y a Ò[`]W\Yfk Y]gY 'b]W\ hg'ZØf'G]Y g]bXž'k Ybb'G]Y i bhYf'GW\k]bXY```Y]XYb'cXYf' < Ò\YbUb[gh'\UVYb°

8]Y 'nk Y]hY'; fi ddY'j cb'?`]ddYbž'X]Y'cVYfYbž'k UfYb'_ÒfdYf`]W\b]W\h'gc'\YfUi gZcfXYfbXž'ZØ\`hYb'g]W\'UVYf'Yl dcb]YfhYf'Ub'\VYgcbXYfg']b'XYb'[fÒ» YfYb'<Ò\Yb"'Jcf'U``Ya 'VY]'nk Y]
HfUj YfgYbUVgW\b]hhYb'ZØ\`hY']W\'a]W\'a Y\f'U`g'j Yfk i bXVUf°
5``YfX]b[g'k i fXYb']b'X]YgYb'GY[a YbhYb'X]Y'VYghYb'5bg]W\hYb
[YZi bXYbž'i bX']W\'k i qghYž'XUqq']W\'b]Y'k Yb][Yf'q]W\Yf'k Uf"

Ÿ\b`]W\k]Y'VY]a '?`YhhYfb'i bX'6ci `XYfb']gh'XUg'di `g]YfYbXY
; YZØ\`'XYf'5i g[YgYhnh\Y]h'i bX'XYf'_ÒfdYf`]W\Yb'< YfUi gZcfXYfi b[
XYg'?`YhhYfbg']a a Yf'bcW\j cf\UbXYb"'6Y]a '?`YhhYfb'i bX
6ci `XYfb']gh'a Ub''YXcW\'bcfa U`Yfk Y]gY'gc']b'XYb'DfcnYgg'XYg
?`YhhYfbg'j Yfh]YZhžXUgg'_Ui a 'NY]h'V`Y]VhžX]Y'5i gg]W\h'ni
[Yb]Y»Yb'cXYf'g]W\'a]h'UbXYfYb'?`YhhYfYfb'Ui gni hUi gW\Yb"

'U`h1 EiYYbghckb'?`YhhYfghY]['DYhYf 25h`Ug'/ '6cchg'8i '_Ubbgh
X]WX 'YXYfnY]h']b'XY]bYa '?`YhhYf[ifh'nifØW_`Y\bYb'ibX'Y]b'GY`Z]Y
a UWXYb°

?`YhhYfb`\Uh`a]f`gW\cb`]a a Yf`GdU» [Ya UW\hž`UVYf`]W\`\UVY`b]W\h X]Y`HYW\b]_`cXYf`X]Y`: À\][_Y]hYbž`i a `X]Y`?`]ddYb`ni `a Y]ghYfbž X]Y`k]f`Ub`X]YgYa `A cf[Yb`[Ybca a Yb`\UVYb"`Bcfa U`Yfk Y]gY k ØfXY`]W\ j]Y```Àb[Yf`VfUi W\Ybž`i a `gc`Ubgdfi W\gj c``Y`?`]ddYb`ni Yf_`]a a Ybž`i bX`]W\`k ØfXY`j]Y``a Y\f`5i gfØghi b[`VfUi W\Yb"

8 Yf'Ei YYbghck b!?`YhhYfghY]['Yfa Ò[`]W\hY'Yg'i bgž'bcfa U`Yfk Y]gY i bdUgg]YfVUfY'?`]ddYb'ni 'Yf_`]a a Ybž'k À\fYbX'k]f'[`Y]W\nY]h][GdU» '\UhhYb'i bX'X]Y'@UbXgW\UZh'[YbcggYb"

5``Yg`]b`Y]bYa 'A cf[Ybk Yf_']b`XYf`5VYbhYi Yf\Ui dhghUXh`XYf K Y`h°

- 'U`h1 2

Ei YYbghck b'J]U': YffUhU. XUg'K YgYbh`]WXY

K Ug. '8 Yb 'E i YYbghck b!? `YhhYfghY]['U`g'HY]` 'Y]bYf'%) !hÀ[][Yb GØX]bgY`!Hci f'j cb'; '5Xj Ybhi fYg'Yf_`]a a Yb"

K c. 18]Y I bhYfVf]b[i b['Ui Z'XYf'Hci f'i a ZUggh'%('BÀW\hY']b
Y]bZUW\Yb'<chY`g'i bX'<cghY`gž'Y]bgW\`]Y»`]W\ 'Y]bYf': Ufa ghUmi bX
Y]bYf'BUW\h']b'Y]bYa 'Y\Ya U`][Yb'; YZ\\hb[b]g"'9]b][Y'I bhYf_\ØbZ\\hY\\\hhYb'; Ya Y]bgW\\UZ\\gV\\XYf'i bX'9\\U[YbVY\hhYb"'K Ybb'\G]Y'Y]b
DUUf'g]bX\\\hat{z}\\YVYb'\G]Y'X]Yg'VY]'XYf'\\hat{6}i W\\i b[\\Yfj cf\(\hat{z}\)XUa]\h'\G]Y
bUW\'A \\hat{0}[\`]W_Y]\h'\]b'8cddY\\Y\\h\\\\UV\\h'\\

K Ubb. '8]Y 'VYghY 'FY]gYnY]h 'ZØf'BYi gYY`UbX']gh 'GYd!5df" '8Yf Gca a Yf 'fBYnYa VYf'V]g': YVfi UfŁ'V]YhYh '\Yfj cffU[YbXYg K UbXYfk YhhYf'i bX'Y]bY'J]Y`nU\`'j cb': Ygh]j U`g'i bX J YfUbghU`hi b[Yb"'8]Y 'BYVYbgU]gcbYb' < YfVgh 'fA Àfn!5dfŁ'i bX : fØ\`]b['fGYd!Bcj Ł'\UVYb'gW\ÒbYg'K YhhYfž'k Yb][Yf A YbgW\Yba UggYb'i bX'b]YXf][YfY'DfY]gY"

K]Y. '8i '_Ubbgh'Y]bYb'Ei YYbghck b!? `YhhYfghY]['ØVYf'XY]bYb'79C j cb'; '5Xj Ybhi fYg'cXYf'X]fY_h'VY]a 'I bhYfbY\a Yb'ØVYf'XYggYb K YVg]hYž'dYf'9!A U]`'Ub']bZc4 WUbmcbYl d`cfYfg"bnž'hY`YZcb]gW\ibhYf'Ž*('''((%''\$\$''cXYf'XifW\'Y]bYb'6Ygi W\']a '6Øfc']a NYbhfi a 'j cb'Ei YYbghck b'i bhYf'Vi W\Yb'' - '@U[YfghfU» Y"

EiYYbghck b'J]U': YffUhU!Hci fYb'VY[]bbYb'Ua '%"'C_hcVYf'&\$%, i bX'`Ui ZYb'V]g']b'XYb'< YfVgh'f5df]`#A U]Ł"'8]Y'G]hni b[Yb'VY[]bbYb nk Y]a U``hÀ[`]W\'i a '-.%) 'I\f'i bX'%'.'\$'I\f'ZØf'%, -'BN8'ff%-I G8Ł'dfc'9fk UW\gYbYa 'i bX'%\$-'BN8'ff+) 'I G8Ł'dfc'?]bXžY]bgW\`]Y»`]W\'HfUbgdcfh'i bX'U``Yf'5i gfØghi b["

K]f'\UVYb'X]Y'GØX]bgY`BYi gYY`UbXg'Ui Z'Y]bYf'%) !hÀ[][YbGØX]bgY`!Hci f'j cb'; '5Xj Ybhi fYg'Yf_i bXYhž'X]Y'%) !hÀ[][Y'FY]gYj cb'K Y``]b[hcb'bUWX'K Y``]b[hcb'ni a 'DfY]g'j cb'&"%(-'; 6D'cXYf&"+-&'l G8'dfc'DYfgcb'ffY'bUWX'8Uhi a Ł'_cghYh"

 $8]Y FY]gYfci hY ZØ hi bhYf UbXYfYa Xi fW XYb 5VY HUga Ub B Uh]cbU DUf_ž: fUbn & gYZi bX E i YYbghck b" 8 Yf DfY]g VY]b U hYh Y]bYb Xi fW [Y\YbXYb 7\]YZ 9 I dYf]YbW C ZZ]W f f7 9 C Ł Y]b Y]bZUW Yg : fØ \ghØ \w_i bX a Y\fYfY A U\ nY]hYb gck]Y nU\ fY]W Y 5_h]j]hAhYb k]Y K UbXYfb ? U \w_ZU\fYb i bX F UXZU\fYb": Øf k Y]hYfY \dark Zcfa Uh]cbYb cXYf 6 i W i b[Yb fi ZYb G]Y $' ((\%+ \% \% \$ Ub cXYf VYgi W Yb G]Y [UXj Ybhi fYg" \w a "$

K]fig]bXij cbi; fc» Vf]hUbb]YbiØVYfiG]b[Udi fii bXi5i ghfu`]YbibUWXBYi gYY`UbXi[YZc[Ybii: Øfi`Àb[YfYi5i ZYbh\ U`hYiUa i7\ Ub[]i5]fdcfh]biG]b[Udi fiYa dZY\ `Ybik]fiXUgiD`UnUiDfYa]i a i5YfchY`iHfUbg]h < chY`iZØfiX]YiYb][YbžiX]YiGWX`UZiVfUi WXYbžii bXiX]YiD`UnUiDfYa]i a @ci b[Yini a i9bhgdUbbYbiUVgY]hgiXYgi; YhØa a Y`gii6Y]XYgi\UVYbk]fik À\fYbXii bgYfYgi%-!ghØbX][Ybi5i ZYbh\ U`hgi[Ybi hnhžigck]YX]Yi_cghYb`cgYiGhUXhfi bXZU\fh°

 $6i \ WXYb \ "G]Y \ ": \ "\emptyset[\ Y \ "ni \ "XYb \ "VYghYb \ "DfY]gYb \ "\emptyset VYf \ "G_mgWUbbYf"$

@cbY`m'D`UbYh'BYk 'NYU`UbX']gh'Y]b'i a ZUggYbXYf'FY]gYZØ\fYf'ZØfBYi gYY`UbXž']XYU`'ZØf'X]Y'Yb][Ybž'X]Y'gckc\`'X]Y'k]W\h][ghYbGY\Ybgk ØfX][_Y]hYb'Yf_i bXYb'U`g'Ui W\'kYb][Yf'VYZU\fYbYGhfU»Yb'kÀ\`Yb'a ÒW\hYb"

CZYb`Y[ib[.'K]f'g]bX'a]h'IbhYfghØhnib['jcb'@cbY`m'D`UbYh'ibX; '5XjYbhifYg'bUW\BYigYY`UbX'[YfY]gh"5``Y'DiV`]_Uh]cbYb'gU[YbXUgž'UVYf'k]f'[UfUbh]YfYb'k]f_`]W\'_Y]bY'dcg]h]jY
6Yf]W\hYfghUhhib["'K]f'gU[Ybž'kUg'k]f'XYb_Yb''[ih'ibX'gW\`YW\h\XUa]h'G]Y'a]h'Y\f`]W\Yf'6YfUhib['ZibX]YfhY'9bhgW\Y]Xib[YbhfYZYb'_ObbYb"

<Ui dhV]`X. 8fYUa gh]a Y

8 YhU1`a

6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. 'Y]bY!fY]gY "XY