

Gc '_ca a Yb 'G]Y 'bUW\': `cfYbWY '6Um'i bX XYb 'i a `]Y[YbXYb '; `Y]gYbž'A U[bYh]W =g`UbX

G]Y ZfU[Yb g]W žk]Y G]Y ni f: `cfYbW 6UmUi ZA U[bYh]W

g`UbX [Y`Ub[Yb3K Ybb G]Y K UbXYfbž5i gg]W hgdi b_hY i bX

GW bcfW Y`b a O[YbžYa dZY\`Y']W =\bYb Xf]b[YbXžY]bYb

XYf _i fnYb K UbXYfk Y[Y ni XYb bcfXOgh`]W Yb 6i W hYb j cb

A U[bYh]W =g`UbX ni bY\a Yb A U[bYh]W =g`UbX]ghY]bYf XYf

j]Y `Yb j YfghY W_hYb GW AhnY 5i ghfU`]Ybg" 9g``]Y[h bi fY]bYb

_i fnYb Gdfi b[a]h XYf: A\fY j cb Hck bgj]``Y YbhZYfbh i bX

]gh gck c\`ni [Ab[`]W U`g Ui W k]X° <]Yf]ghY]b @Y |hZUXYbž

i a Y]bYb a Y]bYf @]YV`]b[gcfhY Ui ZXYf =bgY`ni Z]bXYb/

: `cfYbW 6Umž]bXYa G]Y Y]bY XYf j]Y `Yb _i fnYb

K UbXYfi b[Yb i bhYfbY\a YbžX]Y j cb XYf GhfU» Y Ui g

YffY]W VUf g]bX" U`h1 / Ua d/, &&\$/A U[bYh]W =g`UbX GY[Y`bž

A U[bYh]W =q`UbX 5hhfU_h]cbYbž/ Ua d \(Y \`)`]d/

A U[bYh]W=g`UbX']gh'Y]bYf'XYf'j]Y`Yb'j YfghYW_hYb'GW\AhnY
5i ghfU`]Ybg"'9g'`]Y[h'bi f'Y]bYb'_i fnYb'Gdfi b['a]h'XYf': À\fY'j cb
Hck bgj]``Y'YbhZYfbh'i bX']gh'gck c\``ni [Àb[`]W\'U`g'Ui W\'k]`X''<]Yf
]gh'Y]b'@Y]hZUXYbž'i a 'Y]bYb'a Y]bYf'@]YV`]b[gcfhY'Ui Z'XYf'=bgY``ni
Z]bXYb/: `cfYbW''6Umž']bXYa 'G]Y'Y]bY'XYf'j]Y`Yb'_i fnYb
K UbXYfi b[Yb'i bhYfbY\a Ybž'X]Y'j cb'XYf'GhfU»Y'Ui g'YffY]W\VUf
g]bX"

 $\label{eq:continuous} $$U^h1 = A U[bYh]W=g`UbX`ShhfU_h]cbYbž $$A U[bYh]W=g`UbX`S_h]j]hAhYbž`A U[bYh]W=g`UbX`GcbbYbi bhYf[Ab[Yž] $$A U[bYh]W=g`UbX`GcbbYbi bhYf[Ab[YZ] $$A U[bYh]W=g`$

9fghYbg. K]Ya UbbuWA U[bYh]W=g`UbX_caah

8]Y 5bfY]gY bUW A U[bYh]W=g`UbX]gh gc Y]bZUW k]Y Y]bY (\$!a]bØh][Y : U\fh a]h XYf : À\fY j cb Hck bgj]``Y "

 $8 \ Ug`[\ \varnothing bgh][\ ghY`GYU@]b_!H]W_Yh`_Ubb`cb`]bY`]a ``JcfUi g`VY] $$; YhMci f; i]XY`[\ YVi W\ h`k YfXYbž'k cXi fW\ ``G]Y`Y]bYb`gWUbVUfYb`EF! 7cXY`Yf\U`hYbž'XYb`G]Y`VY]a ``9]bghY][\ Yb`Ui Z`\=\ fYa ``HY`YZcb j Yfk YbXYb`_\ObbYb"$

K]Y '_ca a h'a Ub 'b UW\': `cfYb WY '6 Um'Ui Z A U[bYh]W=g`UbX3

8i ZfU[ghːX]WX žːk]Y ːXi ˈbUWX :: `cfYbWY ·6Um·_ca a gh3·K UbXYfYf \UVYb ˈnk Y] ·A Ò[`]WX_Y]hYbžYbhk YXYf ·Y]bYb ·6i gˈj cb ˈBY``m·6Um·ni bY\a Yb i bX ·XYb ·5fh\i f ·6Um·HfUW_ ·ni ˈk UbXYfb ·cXYf ·j cb <cfgYg\cY ·6Um·Ui gʻni ˈk UbXYfbž′j cfVY] ·Ub ·6U`X]b[ˈi bX ·F UX]WU` 6Um'

: `cfYbWY 6Umj ca 'BY``m'6Um': Yffm'HYfa]bU`

K Ybb'G]Y'X]fY_h'j ca 'BY``m'6Um': Yffm'HYfa]bU``_ca a Ybž'bY\a YbG]Y'Ua 'VYghYb'Y]bYb'6i g'j ca 'HYfa]bU`'ni f'6i g\U`hYghY``Y'H\Y: cfhg"8]Y'?cghYb'ZØf'XUg'6i gh]W_Yh'VYhfU[Yb'' ž+\$'I G8"'\#Wk & ØfXY''YXcW\'Ya dZY\`Ybž'Y]bY'HU[Yg_UfhY'ni '_Ui ZYbž'a]h'XYf'G]Y ni a 'DfY]g'j cb'bi f'nk Y]'H]W_Yhg'i bVY[fYbnh'Ui Z'A U[bYh]W\=g`UbX\Yfi a ZU\fYb'_ObbYb"

Jcb'H\Y': cfhg'Ui g'gc``hYb'G]Y'XYf'FUX]WU``6Um'FcUX'V]g'\]bhYf 5fh\i f'6Um'Zc`[Yb"'9g']gh'i b[YZÀ\f'Y]b'' \$!a]bØh][Yf'GdUn]Yf[Ub[ž UVYf'X]Y'@UbXgW\UZh']gh'Yg'k Yfh"'K Ybb'G]Y'NY]h'\UVYbž'gc``hYb'G]Y i bVYX]b[h'Ui W\']b'5fh\i f'6Um'<U`h'a UW\Yb"

K UbXYfb G]Y Ui Z XYa 6U X]b[ž F UX]WU i bX : `cfYbWY HfUW_ i cb XYf < cfgYg\ cY 6Um Ui g</pre>

8 Y f 'V Y] 'k Y] hYa 'V Y ghY 'K Y [ž'i a 'b U W ': `c f Y b W '6 U m 'n i '[Y `U b [Y b ž] ghž' X Y b '6 U `X] b [ž' F U X] W U `'i b X ': `c f Y b W '6 U m 'H f U W 'j c b ' < c f g Y g \ c Y 6 U m 'U i g 'n i 'b Y \ a Y b "

 $8]Y'K UbXYfi b['Z\emptyset\backslash fh'G]Y'fi bX'i a 'X]Y'VYghYb'6i WkhYb'Ui Z A U[bYh]W=g`UbX'ni a 'G][\hgYY]b['i bX'GWkbcfWkY`b"'9g'[]Vh'Ui WkY]b][Y'i b['Ui V`]WkY'5i gg]Wkhgdi b_hY'Ui Z'XYa 'K Y[<math>\check{z}$ 'X]Y'G]Y'b]Wkh j YfdUggYb'gc``hYb"'8]Y'K UbXYfi b['XUi Yfh'Yhk U'' 'Ghi bXYb'ZØf XYb'<]b!'i bX'F \emptyset W_k Y["

U`h1 ; YcZZfYm'6Um'A U[bYh]W=g`UbXž'5bZU\fh'ni f'; YcZZfYm'6Um A U[bYh]W=q`UbX 2

U`h1 A U[bYh]gW\Yf*bgY`gdUn]Yf[Ub[2

U`h1 2

 $8 \ Yf'K \ UbXYfk \ Y[\ :: \ `cfYbW''6Um'VY[\]bbh'Ua \ `Ogh`]WkYb''9bXY'XYf \\ < cfgYg\ cY'6Umž'[\ `Y]Wk'\]bhYf'XYf'6i g\ U`hYghY``Y'Ua \ `GhfUbX'''9g \\ [\]Vh'a \ Y\ YfYfY'K \ Y[\ k \ Y]gYf'Ybh`Ub[\ `XYg'K \ Y[\ Yg'i \ bX'Yg'gc``hY \\ n]Ya \ `]Wk'Y]bZUWk'ni \ 'Z]bXYb'gY]b'''I a \ `\= bYb'ni \ '\Y`ZYbž'\UVY']Wk \\ XYb'6Y[\]bb'XYf'GhfYW_Y'Ui Z'XYf'? UfhY'i bhYb'a Uf_]Yfh''$

$GY[Y`b`Ui\ Z`A\ U[bYh]W=g`UbX$

±W. `\UVY`A U[bYh]W=g`UbX`Ui Z'a Y]bYa 'nk Y]a cbUh][Yb'GY[Y`hÒfb

j cb A UW_Um ni a $8U]bhfYY F]j Yf Yf_i bXYh" K À\fYbX X]YgYf NY]h Ub_YfhYb k]f ZØf Y]bY K cW Y]b XYf < cfgYg\cY 6Um"$

@9G9B'G=9'85G'G9; 9@@C; .'GY[Y`hU[YVi W\'K cW\Y'+' 'A U[bYh]W = \mathfrak{g} `UbX'

'U`h1 GY[Y`b'Ui Z`A U[bYh]W=g`UbXž'5hhfU_h]cbYb'Ui Z`A U[bYh]W=g`UbXž'5_h]j]hÀhYb'Ui Z`A U[bYh]W=g`UbXž'GcbbYbi bhYf[Àb[Y'Ui Z A U[bYh]W=g`UbX 2

5_h]j]hÀhYb]b : `cfYbW 6Um

6]g': `cfYbWY'6Um'[YgW\UZZh3'5V[YgY\Yb'XUj cbž'XUgg'G]Y'bi f'Ua GhfUbX'YbhgdUbbYbž'Z]bXYb'G]Y'\]Yf'Y]b][Y'Yd]gW\Y'8]b[Yž'X]Y'G]Y bUW\ $A fYf'5b_i bZh'hi b'_ObbYb''$

GW bcfW Y`b

:`cfYbWY'6Um']gh'VYfØ\a h'XUZØfž'Y]bYf'XYf'VYghYb
GW\bcfW\Y`d`ÀhnY'Ui Z'A U[bYh]W=g`UbX'ni 'gY]b"'9g'[]Vh'a Y\fYfY
k Y]»Yž'X]Ua UbhZÒfa][Y'F]ZZa Uf_]Yfi b[Ybž'X]Y'Ui Z'Y]bYb'_`UfYb
GW\bcfW\Y`dZUX'\]bk Y]gYb"

9gʻ]ghʻUa ʻVYghYbžʻVY]ʻ: `i hʻ]bʻXYfʻ: `cfYbWYʻ6Umʻni ʻgWXbcfWXY`bžʻXU YgʻY]bZUWXYfʻ]ghžʻni a ʻF]ZZʻ\]bUi gni [Y\Yb" U`h1 GWXbcfWXY`bʻUi ZʻAU[bYh]W=g`UbX 2

:]bXYbG]YXYb[YY]aYbUV[YY]YbYbGhfUbX

 $k\ Ybb\ G]Y\ a\ X]Y\ Y\ Yb\ G]Y\ Yg\ b\ Yf\ U\ Yb\ GY]hY\ XYf\ 6i\ Wh \\ \ \ Yfi\ a\ [\ Y\ Yb\ Z\ Yff\]W\ Yb\ G]Y\ Yg\]bb\ Yf\ U\ V\ j\ cb\ Yhk\ U\ \&\ A\]bi\ hYb\ "$

G]Y'a ØggYb'Ui Z'Y]b][Y'[fc»Y': Y`gYb'_`YhhYfb'i bX'Ui Z'Y]b][Yb 6Ui a ghÀa a Yb'VU`UbW]YfYbž'k À\fYbX'G]Y'\]bØVYf[Y\Yb"'GcVU`X G]Y'YXcW\'Ub_ca a Ybž'\UVYb'G]Y'Y]bYb'UV[Y`Y[YbYb'Cfhž'XYf dYfZY_h'ni a 'GW\bcfW\Y`b'cXYf'Y]bZUW\'ni a '; Yb]Y»Yb'XYf : `cfYbW''6Um']b'j Ò``][Yf'5V[YgW\]YXYb\Y]h']gh'' U`h1 A U[bYh]gW\Y'=bgY`']b'XYf'6i W\h'j cb': `cfYbn 2

: $U\fYb\G]Y\k Y]hYf\bUWX\5fh\i f\6Um$

K Ybb'G]Y'5fh\i f'6Um'Ui Z'XYa 'K Y['Xcfh\]b'b]W\h'Yfk]gW\h'\UVYbž []Vh'Yg'Y]bY'GhfU» Y'i bX'Y]bYb'DZUXž'X]Y'j ca 'gØX`]W\Yb'9bXY'XYf : `cfYbW'6Um']b'F]W\hi b['5fh\i f'6Um'k Y]hYfZØ\fYbž'X]Y'G]Y bY\a Yb'_ÒbbhYb"

 $: \varnothing f`X]Y \Upsilon b][\Upsilon b \check{z}`X]Y`b UW `< cfg \Upsilon g \ c \Upsilon `6 Um`ni f \varnothing W _ \Upsilon \ f \Upsilon b \check{z}`]gh`\Upsilon g`bi f \Upsilon]b`_i fn \Upsilon f`\&\$!a]b \varnothing h][\Upsilon f`5 V gh \Upsilon W \Upsilon f`ni `\Upsilon]b \Upsilon f`X \Upsilon f`V \Upsilon gh \Upsilon b`6 i W h \Upsilon b X \Upsilon f`\daggraph g \Upsilon`"$

K Ybb'G]Y'a Y\f'j cb'XYb'j YfVcf[YbYb'GW\ÀhnYb'j cb'A U[bYh]W

•g`UbX'Yf_i bXYb'a ÒW\hYbž'XUbb'Ya dZY\`Y']W\' → bYb'Xf]b[YbXž
i a 'X]Y'@UbXni b[Y'\Yfi a 'bUW\'5fh\i f'6Um'ni '`Ui ZYb

< U`hYb G]Y 5i ggWX Ui bUWX ?cU`Ug

 $A U[bYh]W=g`UbX']gh'X]Y'<Y]a Uh'j cb'<ibXYfhYb'j cb'?cU`Ug"'G]YV`Y]VYb'ZØf'g]W\ž'W\]``Yb'UVYf'[YfbY']b'XYb'9i_U`mdhi gVÀi a Yb"6Y\U`hYb'G]Y'X]Y'GhfYW_Y']a '5i[Y"$

8 Yb_Yb^G]Y^XUfUbž'k Ybb^G]Y^g]Y^gY\Ybž'g]bX^g]Y^n]Ya^`]W\gW\ØW\hYfb"; YVYb^G]Y^]\bYb^Yhk Ug^D`Uhn^i bX^VYk i bXYfb^G]Y^g]YUi g^XYf^: YfbY"

U`h1 2

U`h1 AU[bYh]gWXY' ≠bgY`gcbbYbi bhYf[Àb[Y'Ua'6Um'HfUW_2

¸ VYfbUW\ hi b[ga O[`]W_Y]hYb Ui ZA U[bYh]W

=g`UbX

BUW. Zc`[YbX`\UVY`]W\`X]Y`XfY]`VYghYb

, VYfbUW\hi b[ga Ò[`]W_Y]hYb`Ui Z`A U[bYh]W=g`UbX`ZØf

j YfgW\]YXYbY`6i X[Yhg`ni gUa a Yb[YZUggh"`8]Y`@]b_g`ZØ\fYb`ni
6cc_]b["Wca ž`XUg`]W\`dYfgÒb`]W\`]a a Yf`bi hnYž`i a `X]Y`VYghYb
5b[YVchY`ZØf`I bhYf_ØbZhY`ni `Vi W\Yb"

- \ VYfbUW\ hYb'G]Y']a '?cU`U'GUbWhi Ufm 6i b[U`ck'6Um'?cU`U'J]``U[Y''8]YgYg'<cghY`']gh Y]bY'XYf'Ua 'VYghYb'VYk YfhYhYb'[Øbgh][Yb I bhYf_ØbZhY'Ui Z'A U[bYh]W=g`UbX"'9g'\Uh'Y]bYb [i hYb'7\]``!Ci h!6YfY]W\Z'Y]bYb'Dcc`'i bX bUhØf`]W\XX]Y'?cU`Ug'j cf'Cfh"
- 6YghYg'<cghY``a]h'5i gg]Wkh.'6UgY'6UW_dUW_Yfg
 '6UgY'6UW_dUW_Yfg']gh'Y]b'Gmbcbma 'ZØf
 dfY]g[Øbgh][Yg'FY]gYb']b'5i ghfU`]Yb"'8]YgYg
 <cghY``V]YhYh'A Y\fVYhhn]a a Yf'ZØf'i bhYf'')'`
 dfc'BUWkh"8]Y'N]a a Yf'g]bX'Y][Ybh`]Wk'_`Y]bY
 <ØhhYb'i bX'V`]W_Yb'Ui Z'XUg'F]ZZ'X]fY_h'Ua
 K UggYf']b'BY``m'6Um"
- 6YghYg'; ÀghY\Ui g'ZØf'DUUfY.'7GhUm']b'D]Wb]W
 6Um' '9]bY'XYf'dfY]g[Øbgh][ghYb
 I bhYf_i bZhga Ò[`]WX_Y]hYb'ZØf'DUUfY']gh'7ghUmg
]b'D]Wb]W6Um''9g'[]Vh'a Y\fYfY'8cddY`n]a a Yf
 ZØf'k Yb][Yf'U`g'*) '`'dfc'BUWXh''

'U`h1 2

'U`h1 2

...

6Ygi WXYb G]Y i bg Ui Z Y]bY!fY]gY "XY