

D`UbYb'G]Y'Y]bY'K Y`hfY]gY.'*': fU[Ybž'X]Y G]Y'g]WX'ni Yfgh'ghY``Yb'gc``hYb

9g'[]Vh'XfY]'[fi bX`Y[YbXY:: fU[Ybž'X]Y'g]W\''YXYf'ghY``hž k Ybb'Yf'Y]bY'K Y`hfY]gY'd`Ubh'K c\]b'k]```]W\3'K]Y'`Ub[Y a ÒW\hY']W\'[Y\Yb3'K]Y'j]Y``k]fX'Yg'_cghYb3'K À\fYbX'X]Y D`ÀbY'; YghU`h'UbbY\a Ybž'ghY``Yb'g]W\'a Y\fYfY'UbXYfY: fU[Yb.'6fUi W\Y']W\'k]f_`]W\'Y]bY'FY]gYj Yfg]W\Yfi b[3'Gc``]W\'ni gÀhn`]W\Y'GW\i\Y'Y]bdUW_Yb3'K]Y'j]Y``_cghYh'X]YHc``kih]a dZi b[3'f\Už'bY]bž'khZ'\L'9g'[]Vh'bcW\Y]b][Y'UbXYfYk]W\h][Y8]b[Yž'X]Y'G]Y'g]W\'ZfU[Yb'gc``hYbž'kYbb'G]Y'Y]bYKY`hfY]gY'd`UbYb''6YfØW_g]W\h][Yb'G]Y'X]Y'Zc`[YbXYb': fU[YbfYW\hnY]h]['jcf'XYf'5VfY]gYž'ia'b]W\h'bif'ZØf'\LYFY]gYžgcbXYfb'Ui W\'ZØf'U``Ygž'kUg'XUbUW\'_caahž'[ih'UiZ[YghY``hnigY]b'''%''AY]bYb'/Uad\Y\`]d/

9g'[]Vh'XfY]'[fi bX`Y[YbXY': fU[Ybž'X]Y'g]W\'YXYf'ghY``hž'k Ybb'Yf Y]bY'K Y`hfY]gY'd`Ubh.'K c\]b'k]``']W\3'K]Y'`Ub[Y'a ÒW\hY']W\ [Y\Yb3'K]Y'j]Y``k]fX'Yg'_cghYb3

K À\fYbX'X]Y'D`ÀbY'; YghU`h'UbbY\a Ybž'ghY``Yb'g]W\a Y\fYfY UbXYfY': fU[Yb.'6fUi W\Y']W\k]f_`]W\Y]bY'FY]gYj Yfg]W\Yfi b[3 Gc```]W\ni gÀhn`]W\Y'GW\i\Y'Y]bdUW_Yb3'K]Y'j]Y``_cghYh'X]Y Hc``k i h]a dZi b[3'f\Už'bY]bž'k hZ''\L

$$\begin{split} &9g`[\]Vh`bcW\ `Y]b][\ Y`UbXYfY`k\]W\&h][\ Y`8\]b[\ Y\check{z}`X]Y`G]Y`g]W\&`ZfU[\ Ybgc``hYb\check{z}`k\ Ybb``G]Y`Y]bY`K\ Y`hfY]gY`d`UbYb"`6YfØW_g]W\&h][\ Yb``G]Y`X]Y\\ &Zc`[\ YbXYb':\ fU[\ Yb``fYW\&hnY]h][\ 'j\ cf`XYf`5VfY]gY\check{z}`i\ a\ 'b]W\&h`bi\ f`ZØf\\ &\Rightarrow fY`FY]gY\check{z}`gcbXYfb`Ui\ W\&`ZØf`U``Ygž`k\ Ug`XUbUW\&`_ca\ a\ h\check{z}`[\ i\ h\ Ui\ Z[\ YghY``h`ni 'gY]b" \end{split}$$

%" A Y]bYb \sim V _ ØbX][Yb cXYf Y]b GUVVUh]WU`

'U`h1 7fcggfcUXg.'D`Ubi b['Y]bYf'FY]gY'i a 'X]Y'K Y`h 2 HfUi a nY]h

K Ybb'G]Y'VYfY]hg'a Y\fYfY'\J\fY']b'Y]bYa 'I bhYfbY\a Yb'hÀh][
g]bXž'_ObbYb'G]Y'i bhYf'I a ghÀbXYb'Y]b'GUVVUh]WU`'Y]b`Y[Yb"'\d
Y]b][Yb': À``Yb'k]fX'X]Yg']a 'GhUbXUfXUfVY]hgj YfhfU['Ub[YVchYb"'\d
UbXYfYb': À``Yb'a ØggYb'G]Y'Y]bYb'VYgcbXYfYb': U```XUZØf
YfghY``Yb", VYf`Y[Yb'G]Y']b'YXYa ': U``ž'cV'G]Y'Y]b'GUVVUh]WU`
bY\a Yb'cXYf'[Ubn'Ui Z\OfYb'a OWXhYb"

 $8 \ Ug^{5}i \ Z^{0}fYb^{7}y^{1}hYh^{2}Y^{1} h^{3}a \ Uh]j \ Y^{1}i^{1}h^{3}h^{1}K \ Ybb^{1}G]Y^{1}h^{4}h^{1}K \ Ybb^{1}G]Y^{2}h^{4}h^{1}K \ Ybb^{1}G]Y^{2}h^{4}h^{1}G]Y^{2}h^{2}h^{1}G]Y^{2}h^{2}h^{1}G]Y^{2}h^{2}h^{1}G]Y^{2}h^{2}h^{1}G]Y^{2}h^{2}h^{2}G]Y^{2}h^{2}h^{2}G]Y^{2}h^{2}h^{2}G]Y^{2}h^{2}h^{2}G]Y^{2}h^{2}h^{2}G]Y^{2}h^{2}h^{2}G]$

&"'FcibX!h\Y!Kcf`X!H]W_Yh'cXYf'DUm'Ug'mci [c3

 $6Y]^5h^0Ug^7 - 6cchg^0VYj - cfni - [Yb^0k]^f^0Um^1Ug^1mci + [c!FY]gYb^2^XU^0Yg - X]Y^A O^[^]W^2 - Y]h^0cZZYb^0Agghz^0Ui - ZY]b^06cch^0ni - [gdf]b[Yb^2^]b^0YhnhYf - Yb^0Agghz^0Ui - ZY]b^06cch^0ni - [gdf]b^0Agghz^0Ui - ZY]b^0Agghz^0Ui - ZY]b^0Agghz^0Agghz^0Ui - ZY]b^0Agghz^0Ui - ZY]b^0Agghz^0Agghz^0Ui - ZY]b^0Agghz^0Agghz^0Agghz^0Ui - ZY]b^0Agghz^0Agghz^0Agghz^0Agghz^0Ui - ZY]b^0Agghz^0Agg$

A]bi hY 'Y]bYb 'F cUXhf]d 'ni 'i bhYfbY\a Yb 'cXYf'g]W\ 'ZØf'Y]bY 'K Y]`Y]f[YbXk c 'b]YXYfni `UggYb"': Øf'i bg'X]Y 'a]h'5VghUbX]bhYfYggUbhYghY '5fh'ni 'fY]gYb"

' "'K]Y]gh'a Y]bY 6cb]hÀh3

 $\label{eq:linear_norm} Nk\ Y]hYbg`_Ubb`=\ fY`?fYX]hk\ \varnothing fX][\ _Y]h`VYY]bhfàWkh][\ h`k\ YfXYbž\ k\ Ybb`G]Y``Ub[\ Zf]gh][\ `c\bY`ZYghYb`K \ c\bg]hn`fY]gYb"`GhY``Yb`G]Y g]Wk\ Yfž`XUgg`G]Y`g]Wk\ `Z\@f`X]Y`Gh]a a UV[\ UVY`Xi\ fWk\ Y]bYb 6Yj\ c``a\ AWkh][\ hYb`fY[\]ghf]YfYb`i\ bX`=\ fY`FYWkbi\ b[\ Yb`i\ bX\?cbhcUi\ gn\@[\ Y`Ub`Y]bY`5XfYggY`gYbXYbži\ a\ `=\ fY\ :\]bUbni\ bhYf`U[\ Yb`gc`cfXYbh`]Wk\ k\]Y`a\ O[\`]Wk\ni\ `\U`hYb"$

("Aigg]WX Z]hhYfk YfXYb3

`U`h1 @Ui ZYbXY`: fUi .`D`Ubi b[`Y]bYf`K Y`hfY]gY 2 HfUi a nY]h

K Ybb 'G]Y 'Y]bY 'K Y`hfY]gY 'd`UbYbž'\UVYb 'G]Y 'k U\fgW\Y]b`]W\ 'Y]b dUUf'Di b_hY 'Ui Z'\Rightarrow\family f'@]ghYž'X]Y 'Y]b '[Yk]ggYg 'A U» 'Ub':]hbYgg YfZcfXYfb 'fh" '6" 'A UW\i 'D]WW\i 'hfY___Ybž'?]`]a UbXgW\Ufc VYghY][Ybž'XYb 'Y]bYb 'c XYf'UbXYfYb 'J i `_Ub 'VYnk]b[YbŁ" '8 Yb_Yb G]Y ']a 'J cfUi g'XUfØVYf'bUW\ 'i bX 'ØVYf`Y[Yb 'G]Yž'k]Y 'Z]h'G]Y 'gY]b a ØggYb" '\Lib 'HfUj Y`g'[UV 'A]W\UY` '7 f]W\hcb 'ni ž'?]`]'Ub[Y[f]ZZYb 'ni \UVYbž'k À\fYbX'Yf'YfVÀfa `]W\ 'gW\`YW\hj cfVYfY]hYh'k Uf" '9f i bhYfgW\ÀhnhYž'k]Y 'Z]h'Yf'gY]b 'a i gghY 'i bX 'YbXYhY 'a]h'6`UgYbž 9fgW\`OdZ\b['i bX'Y]bYf'@Ugh'ZØf'gY]bY'6Y[`Y]hYf"

 $5 \text{ fVY}] \text{hYb'G} \text{Y'gc'Zf} \text{Ø} \text{`k'} \text{Y'a'} \text{O}[\text{`}] \text{W. TUb'} \Rightarrow \text{fYf': }] \text{hbYggž'i a'b}] \text{W. h'bif } \text{Ubgdfi W. gj c``Y'5_h]j } \text{hAhYbž'gcbXYfb'Ui W. 'XYb'FY} \text{gYU``hU}[\text{cdh}] \text{a U``[Yb]Y} \text{Yb'ni '_ObbYb''K Ybb'G}] \text{Y'Z} \text{h'g}] \text{bXž'} \text{gh'Yg'k Yb} \text{[Yf UbghfYb[YbXž'`Ub[Y'GhfYW_Yb'ni '[Y\Yb'i bX'] \Rightarrow \text{fYb'Fi W_gUW_ni hfU}] Yb''}$

) " 6fUi WXY]WX Y]b HYghUa Ybh3

 $K \]f'k \]ggYb \ \check{z}'XUgg'Yg'Y]b'V]ggWXYb'a \ cfV]XY']gh\check{z}'UVYf'Y]b \\ HYghUa Ybh'ni \ gWXfY]VYb']ghYY]bY', VYf'Y[ib['k Yfh\check{z}'k Ybb'G]Y'Y]bY \\ Ui g[YXY\bhY'K Y'hfY]gY'd `UbYb'''8 Ug'FY]gYb']b'X]Y'a Y]ghYb'HY]'Y \\ XYf'K Y'h']gh'k U\fgWXY]b`]WX'[YbUi gc'g]WXYf'k]Y'XUg'@YVYb']b \\ @cbXcb \check{z}'UVYf'Yg']gh'j YfbØbZh][\check{z}'XUg'GWX']a a ghY'ni 'd`UbYb'''C\bY Y]b'HYghUa Ybh'k]fX'XYf'BUWX`Ugg'Y]bYf'DYfgcb'[Ya À»'XYa ; YgYhn'Ui Z[YhY]'hž'k Ug'a Õ[`]WXYfk Y]gY'b]WXh']\fYb'K ØbgWXYb Ybhgdf]WXh'''GWXfY]VYb'G]Y'Y]bYž'i a '\Rightarrow fY'@]YVYb'ni 'gWX ØhnYb'''8]Yg _Ubb'\@VYf'Y]bYb'FYWXhgUbk U'h'cXYf'Y]bYb'GdYn]U`]ghYb'ZØf HYghUa YbhgYfghY``i b['YfZc`[Ybž'cXYf'G]Y'_ObbYb'Yg'gc[Uf'gY'Vgh hi bž'k Ybb'X]Y'9fVhY]`i b['YfZc`[Ybž'cXYf'G]Y'_ObbYb'Yg'gc[Uf'gY'Vgh hi bž'k Ybb'X]Y'9fVhY]`i b['YfZc`[Ybž'cXYf'G]Y'_ObbYb'Yg'gc[Uf'gY'Vgh hi bž'k Ybb'X]Y'9fVhY]`i b['YfZc`[Ybž'cXYf'G]Y'_ObbYb'Yg'gc[Uf'gY'Vgh$

* "'K Ug'Yfk UfhY']WX j cb'a Y]bYf'FY]gY3

 $6 \label{eq:cf-G-Y-Y-B-Y-K-Y-hfY-g-Y-G-Y-K-Y-hfY-g-Y-K-Y-hfY-g-Y-K-Y-hfY-g-Y-K-Y-hf-Y-g-Y-K-Y-hf-Y-g-Y-K-Y-hf-Y-g-Y-K-Y-hf-Y-g-Y-Y-hf-Y-Y-g-Y-Y-hf-Y-Y-g-Y-Y-hf-Y-Y-g-Y-Y-hf-Y-Y-g-Y-Y-hf-X-Y-f-K-Y-Y-hf-X-Y-f-K-Y-Y-hf-X-Y-f-K-Y-Y-g-Y-Y-hf-X-Y-f-W-Y-Y-hf-X-Y-hf-X-Y-f-W-Y-Y-hf-X-Y-hf-X-Y-f-W-Y-Y-hf-X-Y-hf-X-Y-hf-X-Y-f-W-Y-Y-hf-X-Y$

K Ybb'G]Y']a 'JcfUi g'ØVYf' \Rightarrow fY'N]Y`Y'bUWXXYb_Ybž'_ÒbbYb'G]Y' \Rightarrow fY 9fZU\fi b['j cb'Y]bYf'`i gh][Yb'K Y`hi a fi bXi b['ni 'Y]bYf `YVYbgj YfÀbXYfbXYb'FY]gY'a]h'XUi Yf\UZhYb'9f[YVb]ggYb a UWXYb"

J]Y`; `ØW_°

 $\label{eq:continuous} $K_Y]hYfY^<]^2Y^VY]^XYf^D`Ubi_b[^Y]bYf^K_Y`hfY]gY^Z]bXYb^G]Y^]b^8cb_h$

C ZZYf'DUdUmUž'i bgYfYa '@Y]hZUXYb'ZØf' \Rightarrow fY'YfghY'FY]gY'i a 'X]Y K Y`h"

@Y]hV]`X.`8fYUa gh]a Y

•

•

8 Y hU]`g

6Ygi WXYb G]Y i bg Ui Z Y]bY!fY]gY "XY